Structural Geology & Resources 2022
16 October 2022

Mineral systems as chemical reactors with no mathematics

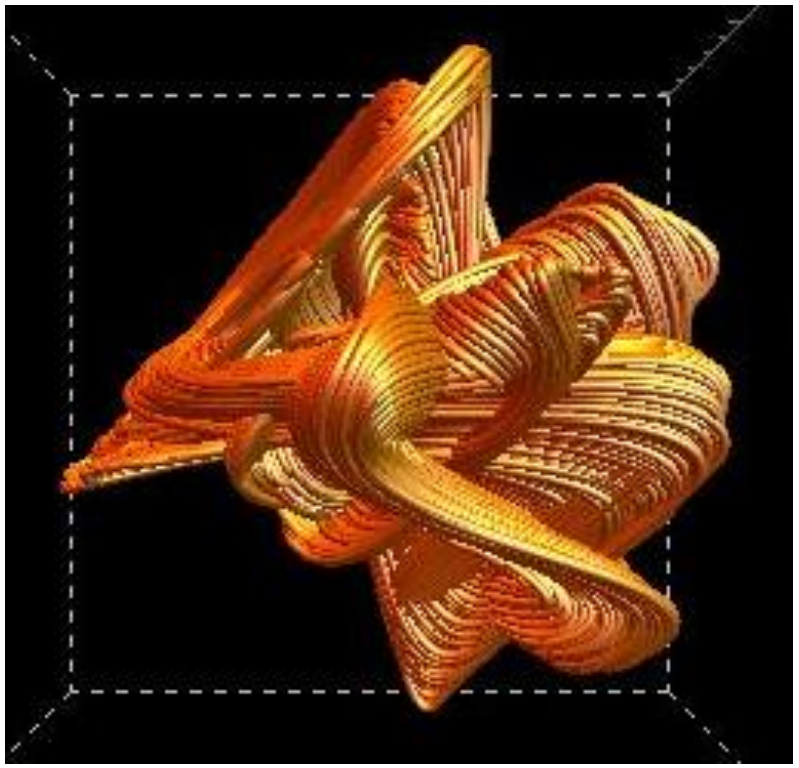
Bruce Hobbs and Alison Ord

Session 4. **10.45 – 12.30**

The nonlinear toolbox: What do we do with all these data?

- **Wavelets and multifractals. Long range correlations.**
- **Recurrence.**
- **Probability distributions**

Fingerprint for a mineralising system



so what do we do with all these data?

Classical statistics

Assumption – statistics are Gaussian

$\Rightarrow$ Mean and variance have a statistical meaning

Assumption – variables involved are "random" and "independent"

- that no correlations exist within the data

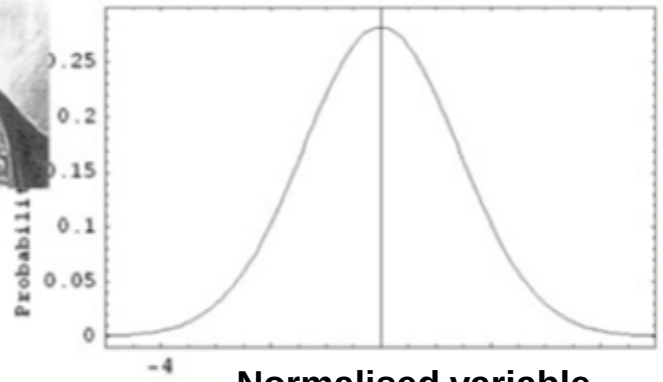
However –

Most data sets 'fractal' in nature

Long range correlations exist

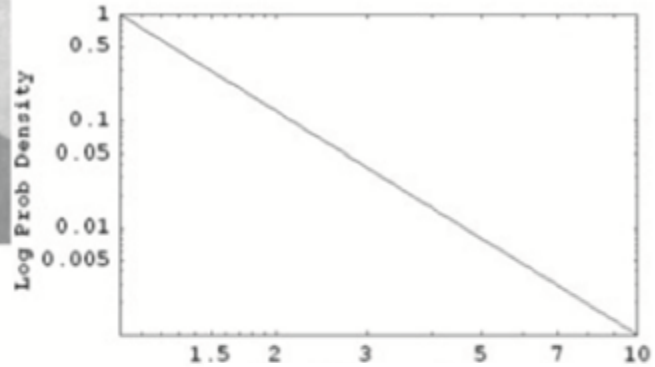
Concepts of mean and variance statistically meaningless

Gauss; bell curve (circa 1800)



Normalised variable

Pareto; inverse power law (circa 1900)



Logarithmic variable

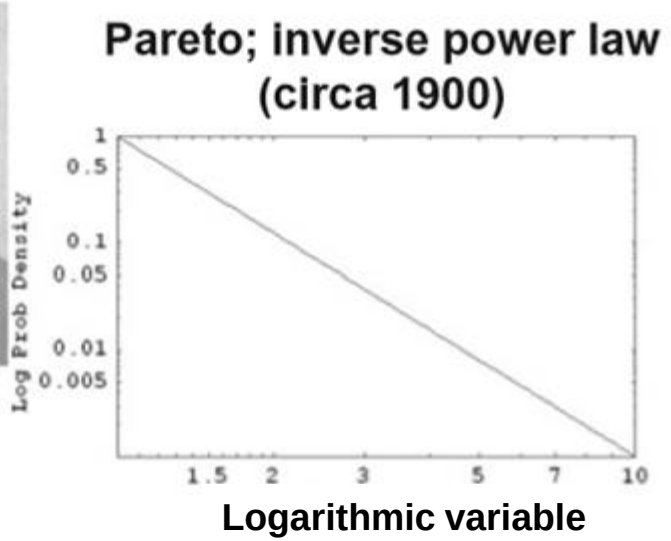
The classical way of treating geological systems

- **Simple scientific world view**
  - linear; output is proportional to input
  - additive
  - simple rules yield simple results
  - stable
  - predictable
  - quantitative
  - normal distribution
  - **no long range correlations**
  - **mean and standard deviation have statistical meanings**

The nonlinear way of treating geological systems

- **Complex scientific world view**
  - non-linear; small changes may diverge
  - multiplicative
  - simple rules yield complex results
  - unstable
  - limited predictability
  - qualitative plus quantitative
  - inverse power-law distributions
  - **long range correlations**
  - **for 2D data, mean and standard deviation are infinite**

Some important words of caution



The term fractal as used in geology can mean two things:

1. The data follow a power law which means if we plot the log of the data against the log of the size we get a straight line.

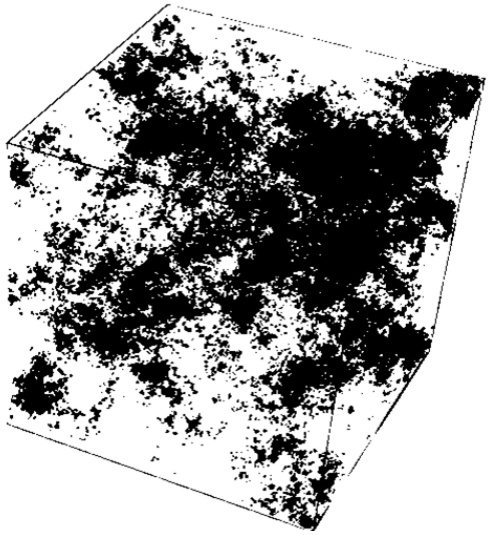
2. The data look the same at all scales; ie the data are scale invariant.

In fact straight line plots are extremely rare and people dream up all sorts of reasons why they are not straight lines.

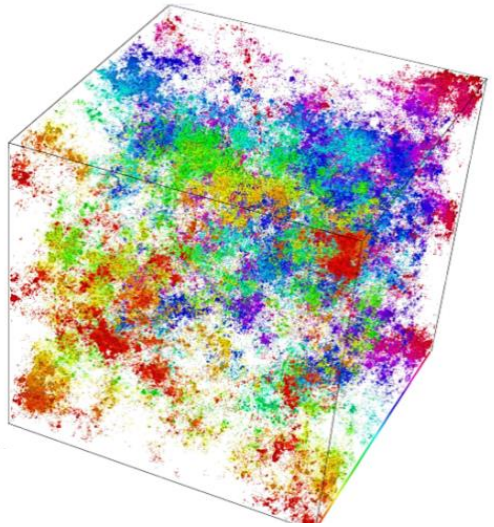
The reason data look the same at all scales is because the data follow the same probability distribution at all scales. It is the probability distribution that is scale invariant, not the data. This results from the same processes operating at all scales.

Most geological data have probability distributions that have long or fat tails which are power law like.

We use the term fractal to mean the degree to which an object fills space.



Fractal
Fills about 2.6 of 3D space

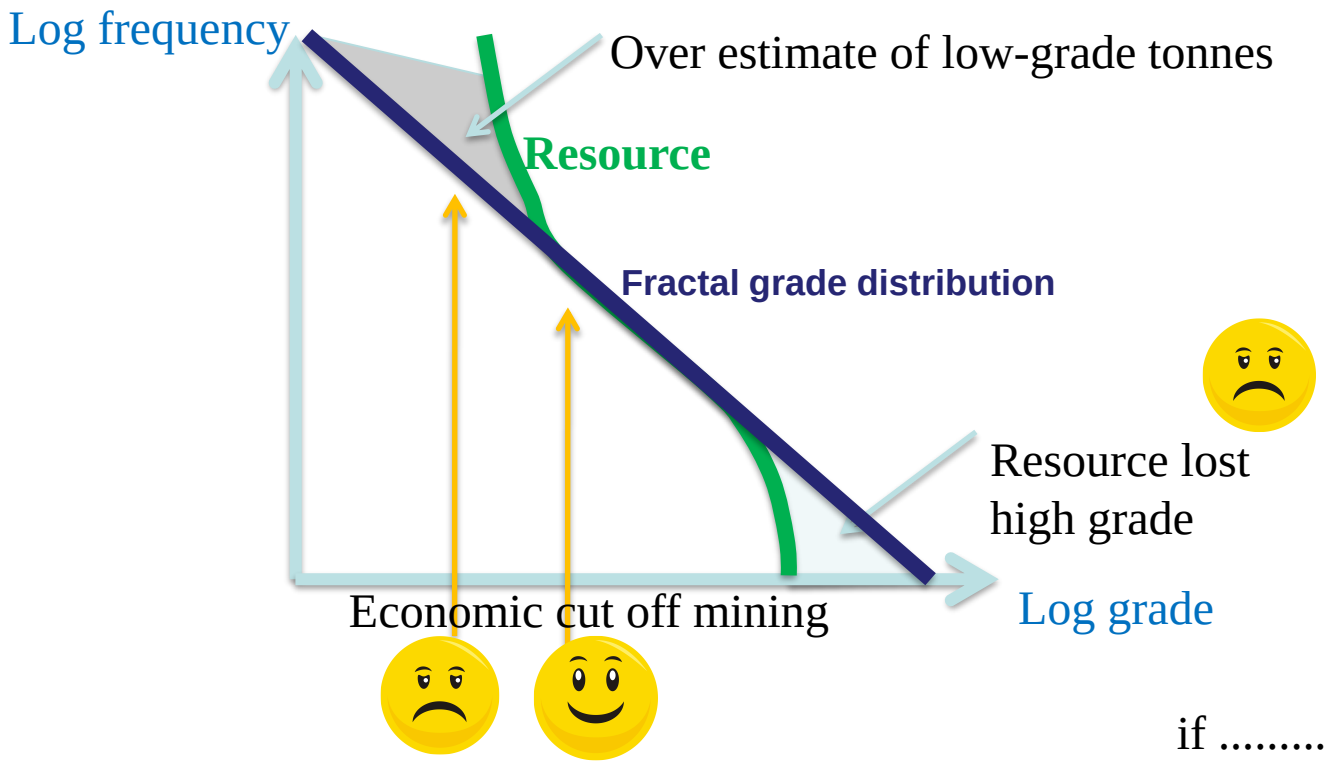


Multi Fractal
Each component fills space to a different degree

If we go Gaussian, we miss all the tails

which is a pity because that's where all the good stuff is.

If ore grade is fractal or fractal-like



if .....

Thank you Julian Vearncombe

Another measure that we would really want when we come to prediction is some measure of periodicity.

Most engineers look immediately to the Fourier transform (FT) (which is easily executed in Excel)

Let us look at the Fourier transform of some quasi-periodic signals and a deterministic chaotic signal.

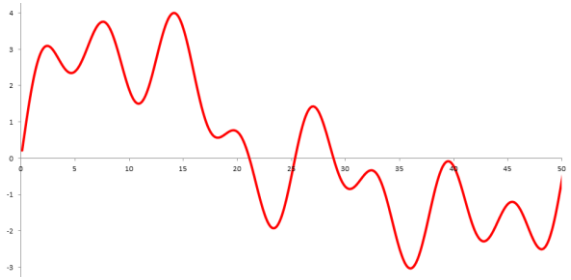
A quasi periodic signal is one that looks to be periodic but never repeats itself.

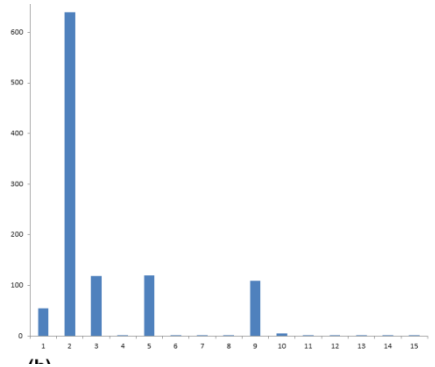
An example is

$y = A\sin(ax) + B\sin(bx)$

where the ratio a/b is an irrational number such as π, 1/6 or sqrt(2).

Quasi periodic

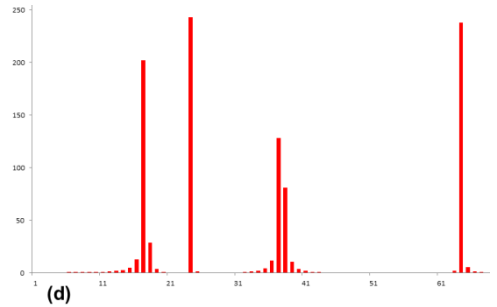




y=sin(x)+sin(x/2)+sin(x/4)+sin(x/6)+sin(x/8)+sin(x/10)

Fourier transform

Quasi periodic

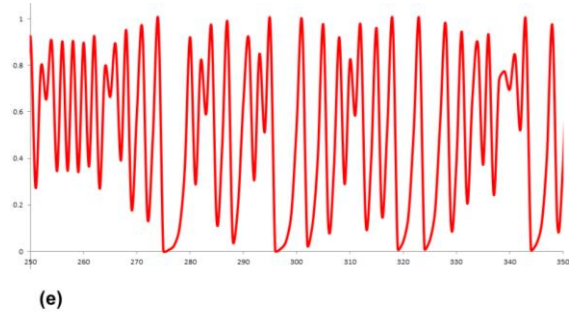


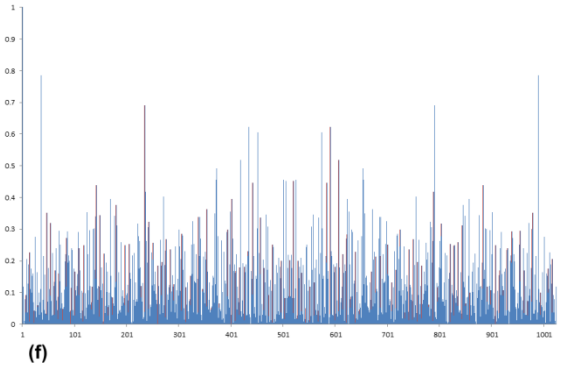
$$y = \sin(x) + \sin(\sqrt{2}x) + \sin(\sqrt{5}x) + \sin(\sqrt{15}x)$$

Fourier transform

The Fourier transform is of great use for deterministic, periodic or quasi-periodic signals

Logistic

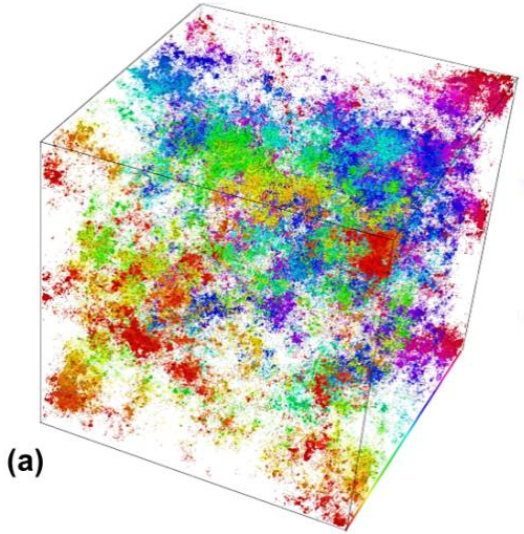




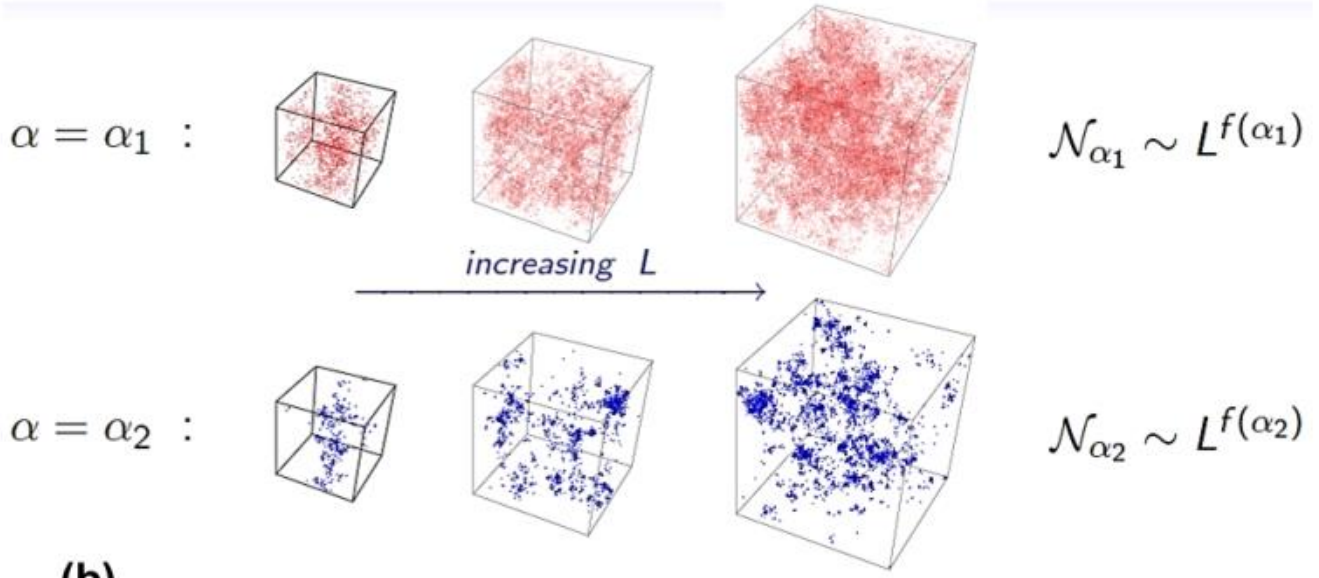
The Fourier transform is of little use for deterministic, chaotic signals

Fourier transform

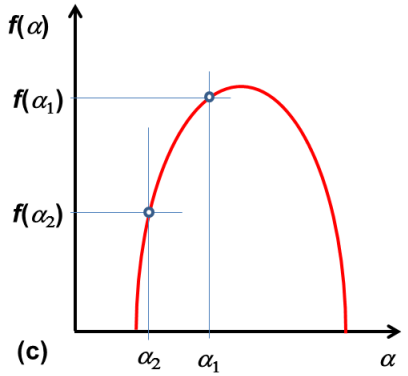
Multifractals; an intertwined set of fractals



(a)



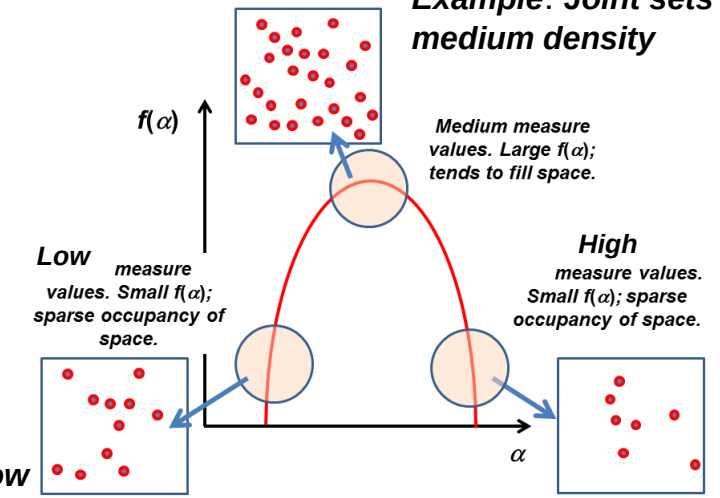
(b)



(c)

***Example**: Joint sets with low density*

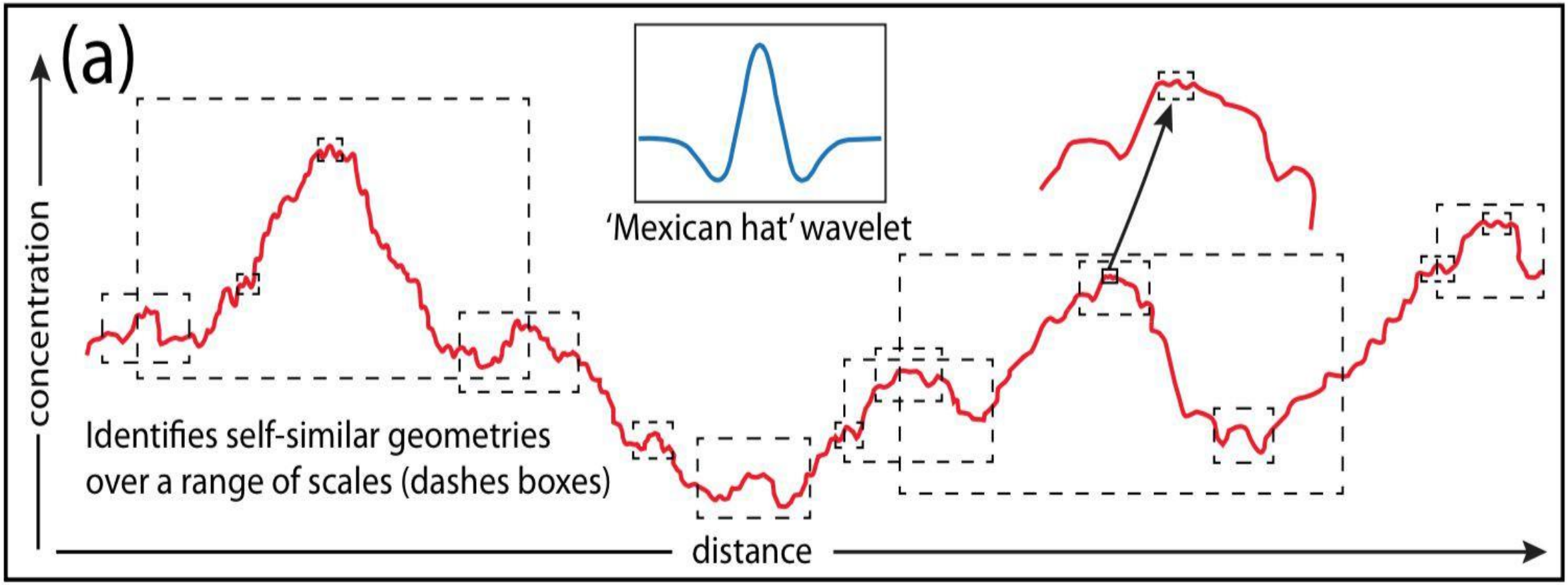
***Example**: Joint sets with medium density*



(d)

***Example**: Joint sets with high density*

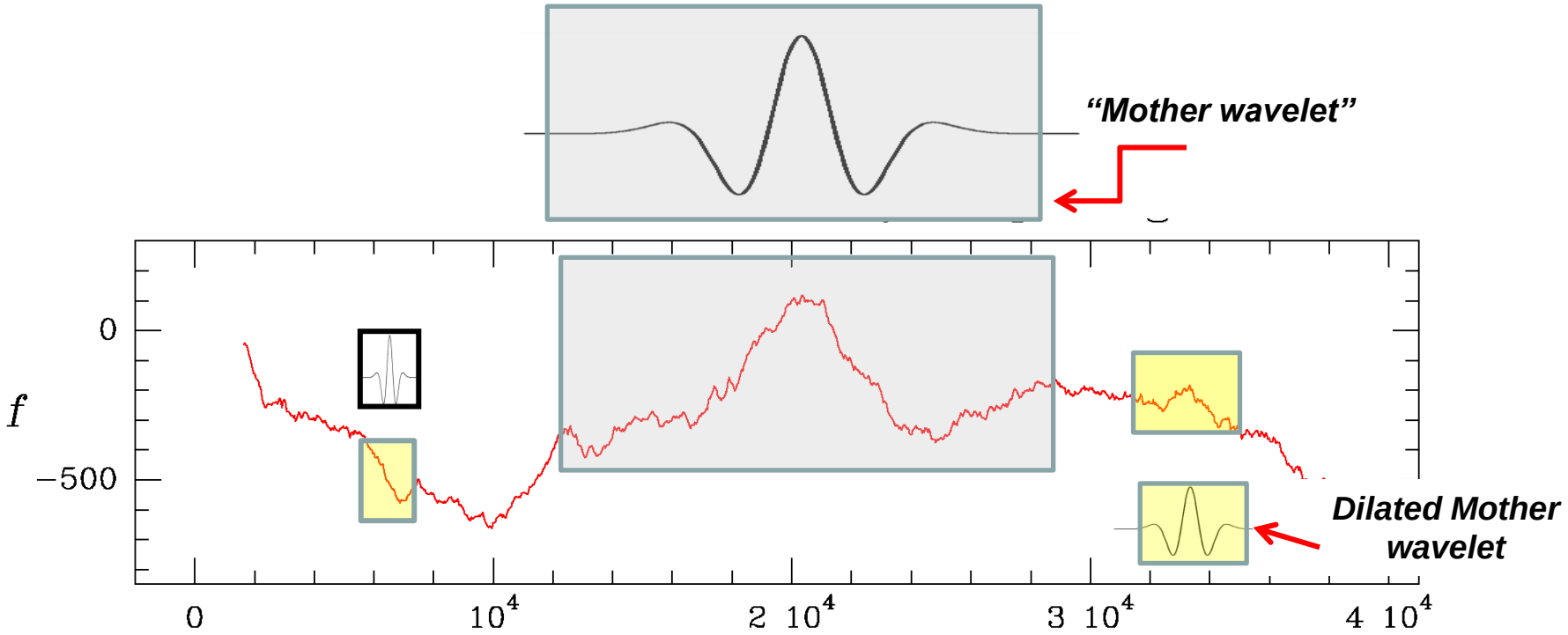
In order to establish if a geometry is multifractal and to quantify the geometry we use a *wavelet transform*



There are all sorts of wavelets, each useful for different patterns.

All of them are localised wave packets of some kind.

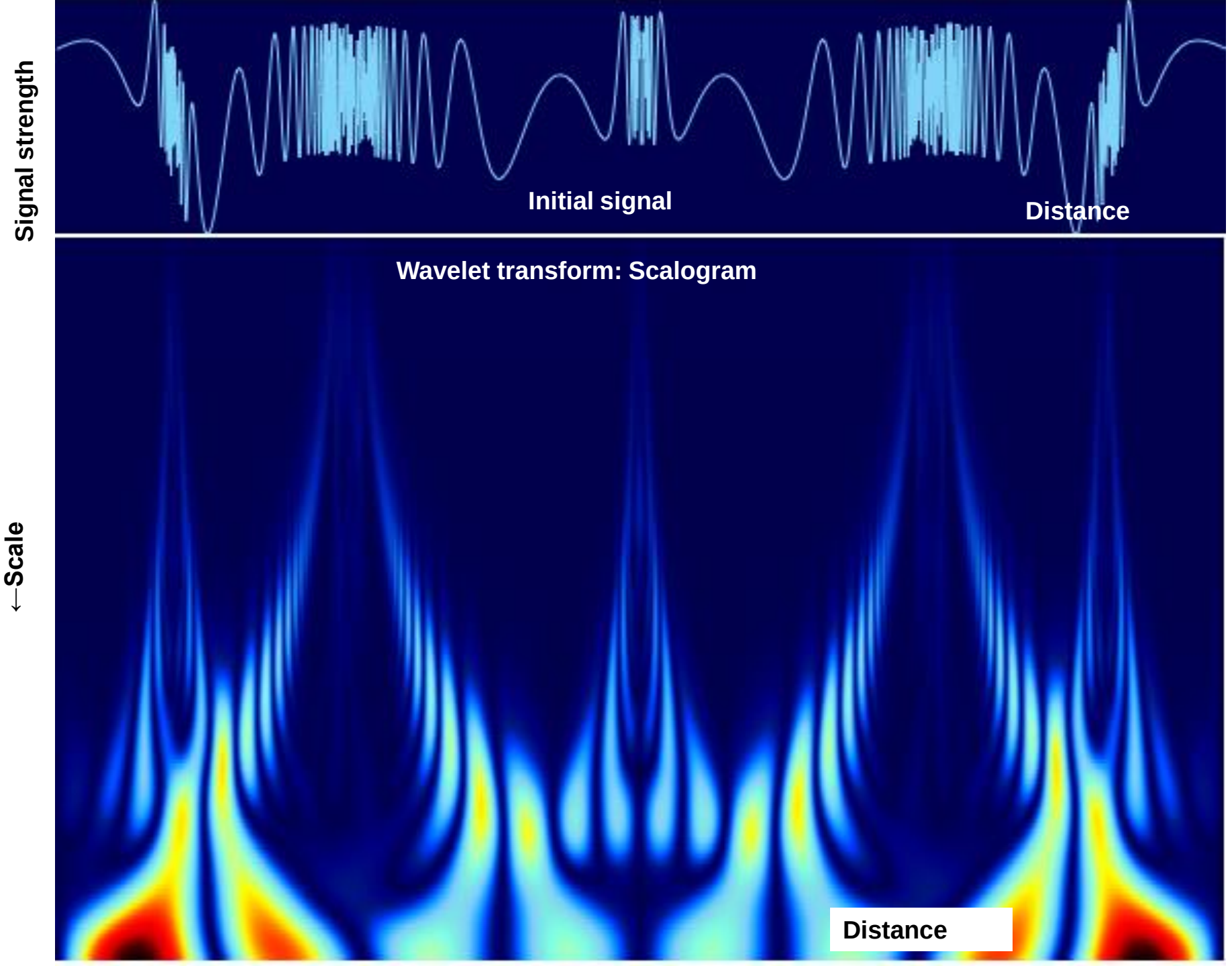
WAVELET ANALYSIS OF FRACTAL SIGNALS

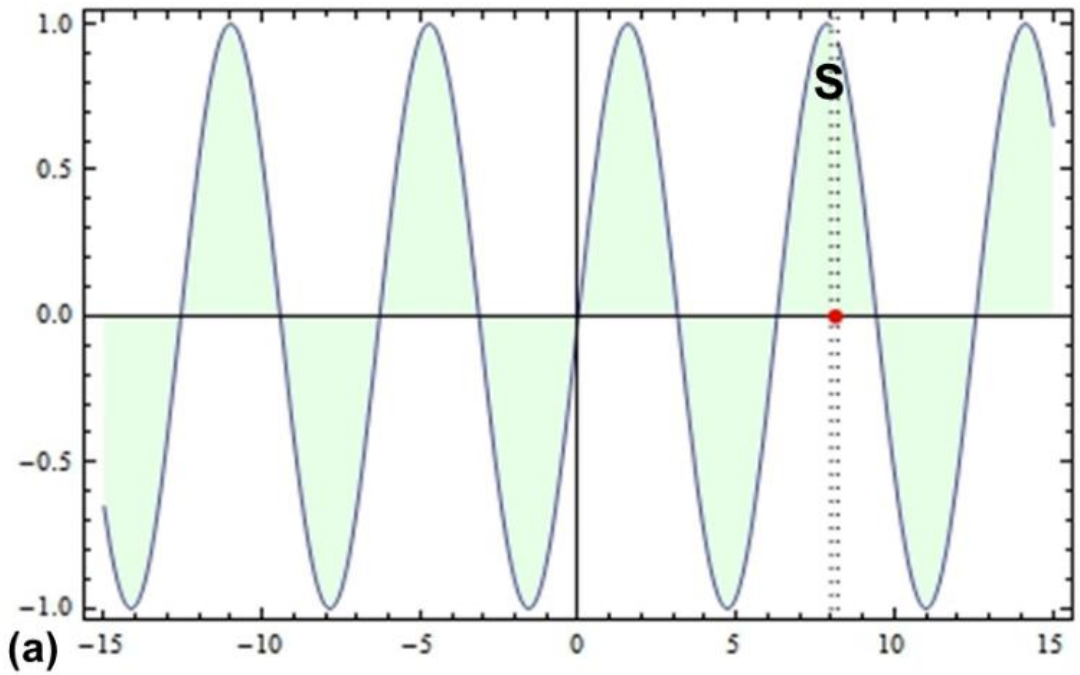


Scan the wavelet across the image with different magnifications and see how closely the wavelet matches the image at a particular scale.

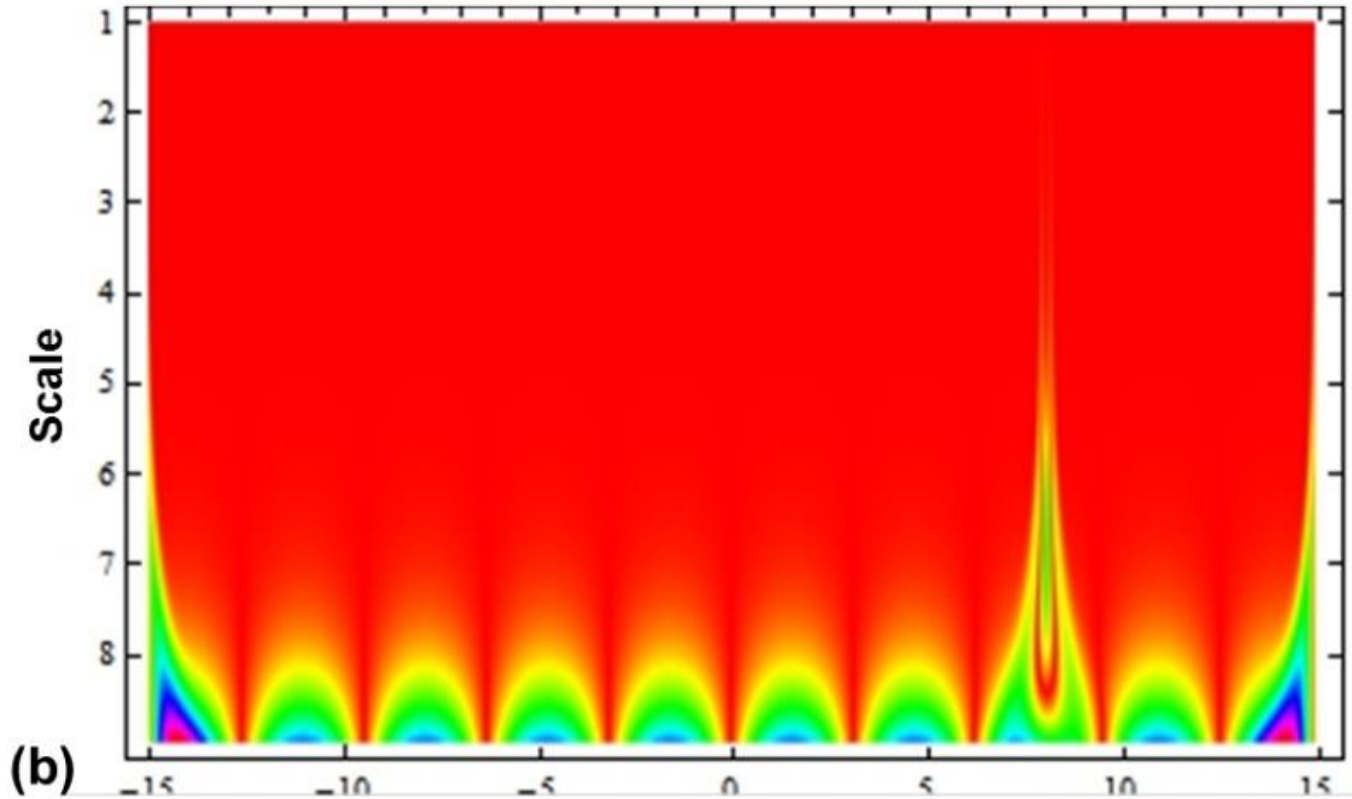
A mathematical microscope.

A generalised box counting procedure.

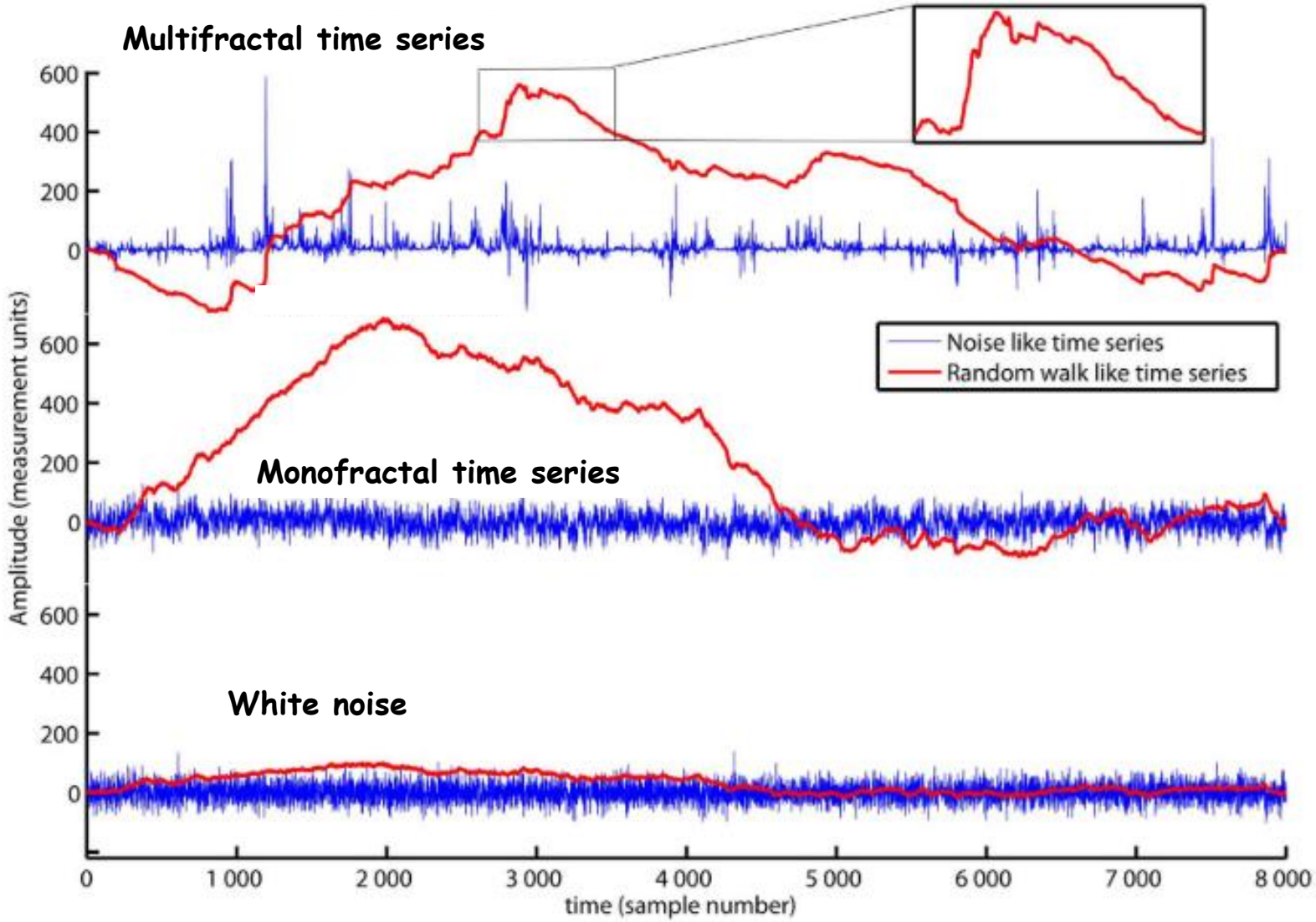

Signal strength
Initial signal
Distance
Wavelet transform: Scalogram
←Scale
Distance


1.0
0.5
0.0
−0.5
−1.0
−15
−10
−5
0
5
10
15
S

(a)

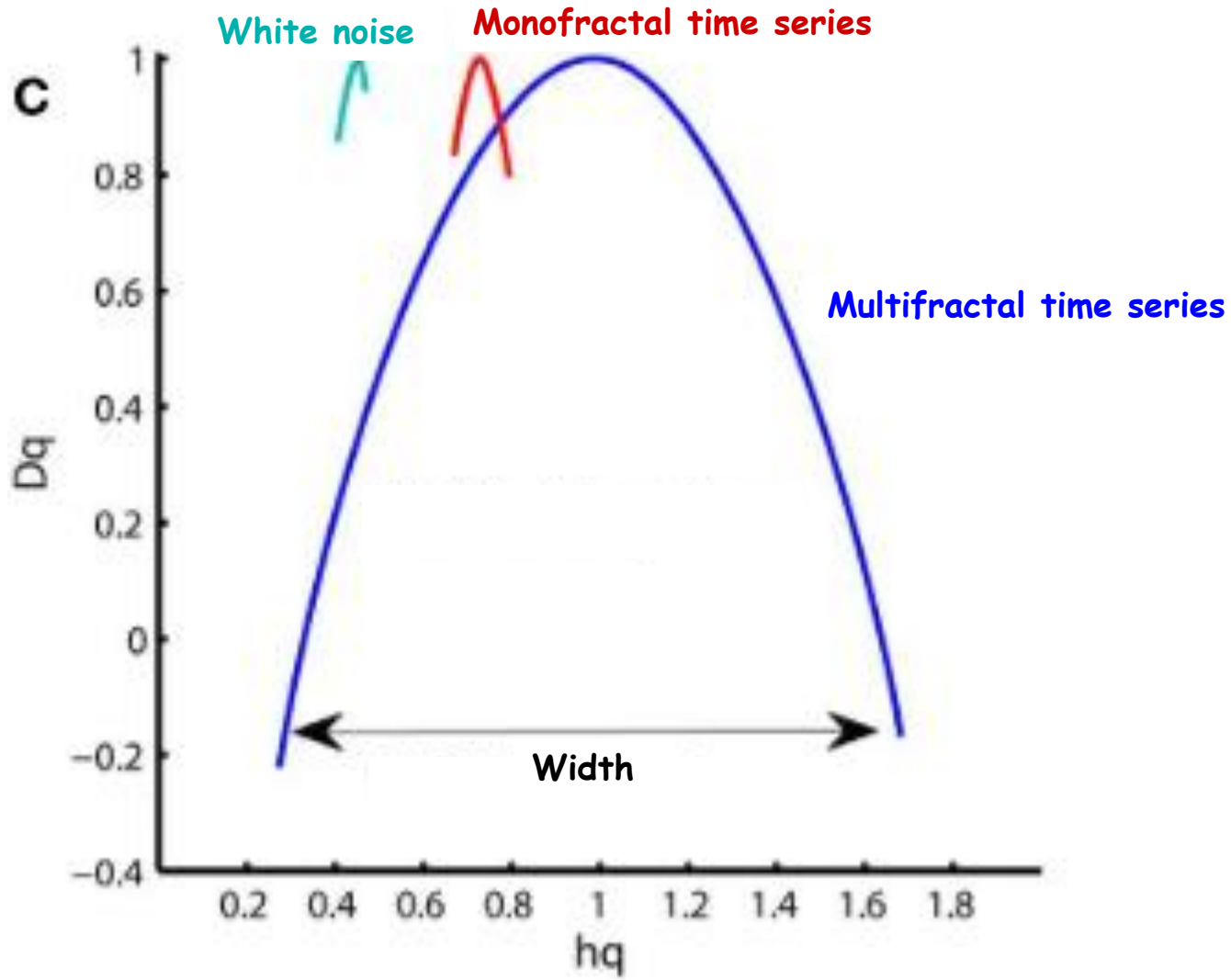

Scale
1
2
3
4
5
6
7
8
−15
−10
−5
0
5
10
15
Distance

(b)

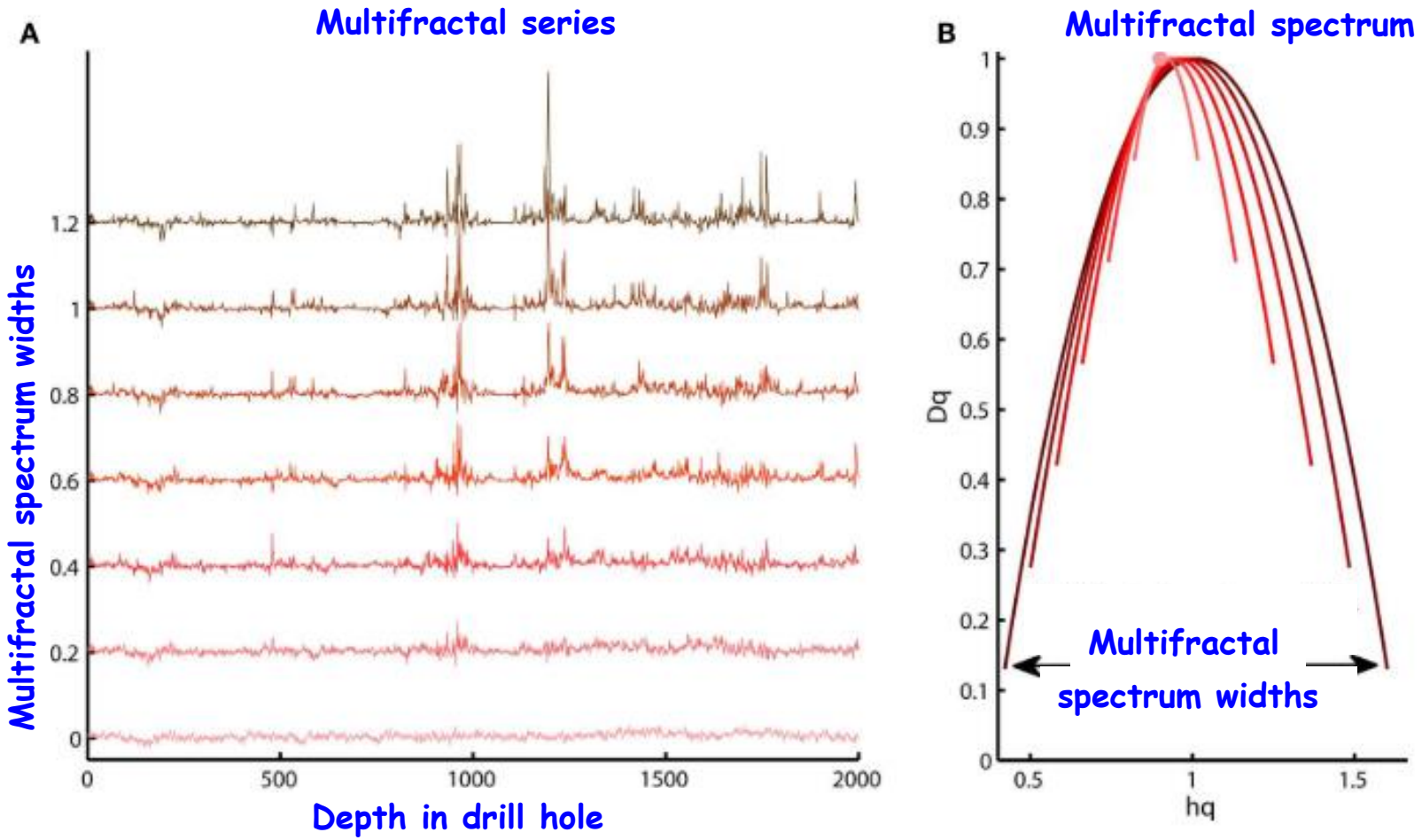


Note the fluctuations in the multifractal time series which are not present within the monofractal time series or the white noise. It is the structure of such fluctuations that we aim to quantify.

Ihlen 2012

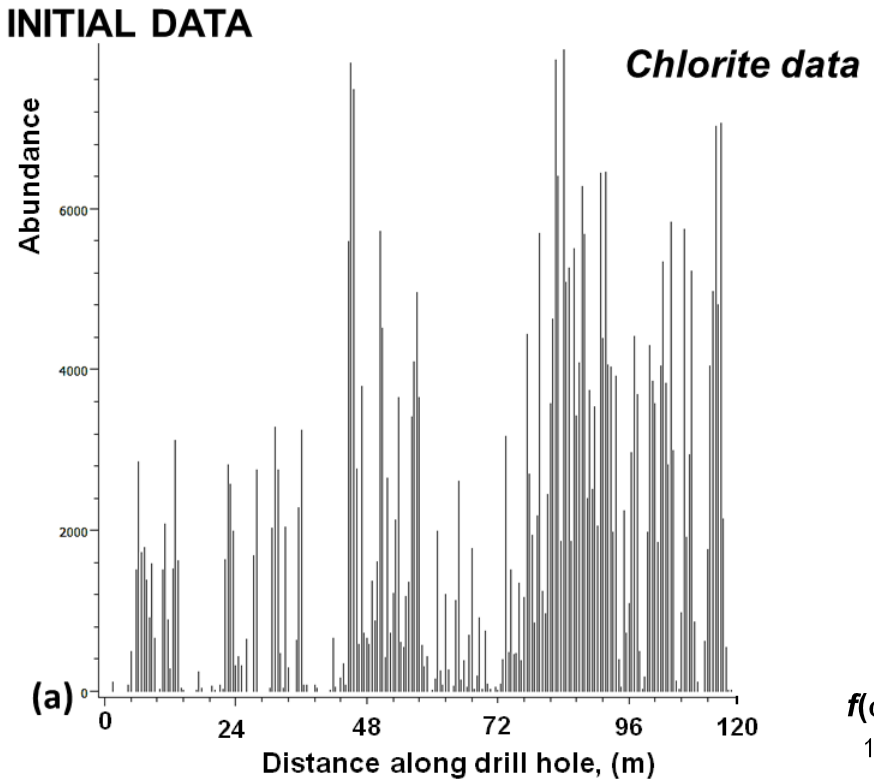

White noise
Monofractal time series
C
Multifractal time series
Width
Dq
hq
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

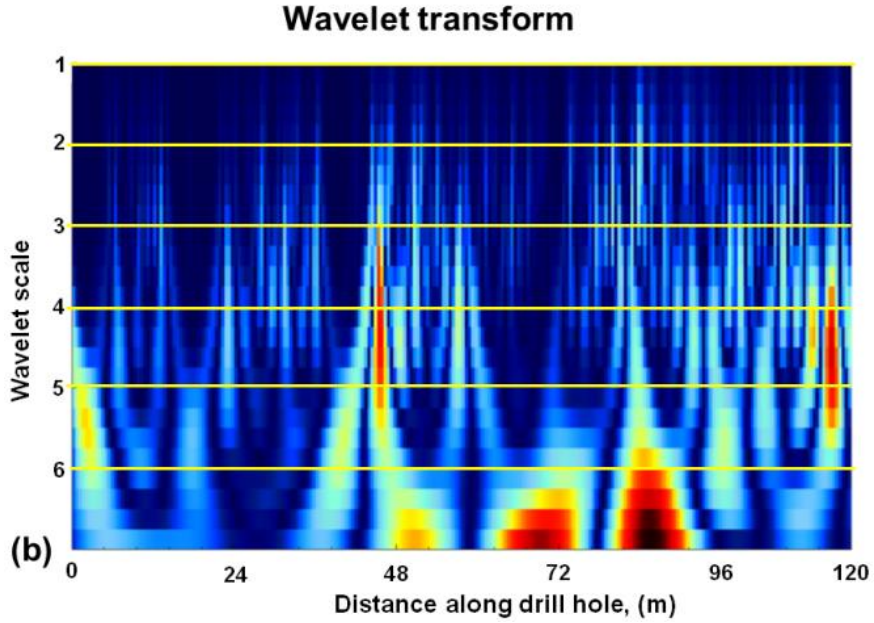
Ihlen 2012



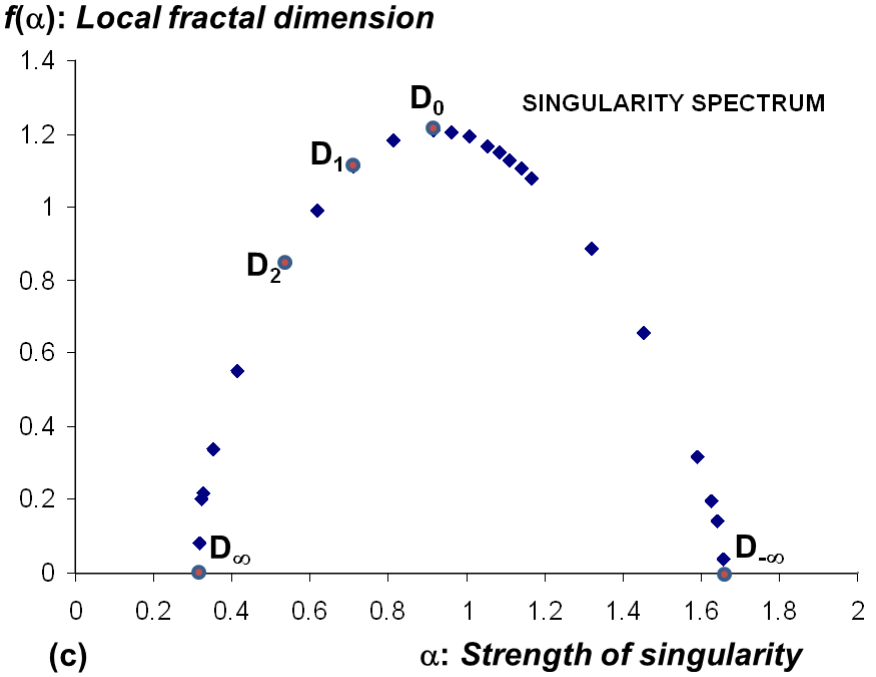
Note how the width of the spectrum increases as the fluctuations increase

Ihlen 2012





The progression from raw data to wavelet transform to singularity spectrum



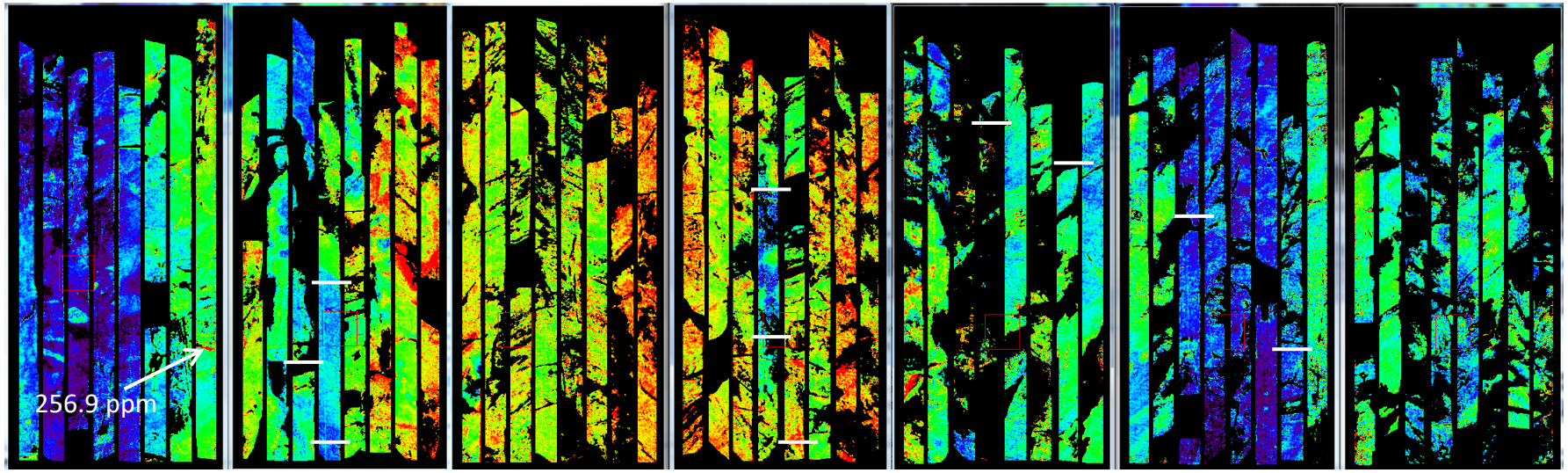
The Data

Mineralogy / Chemistry

Sericite composition

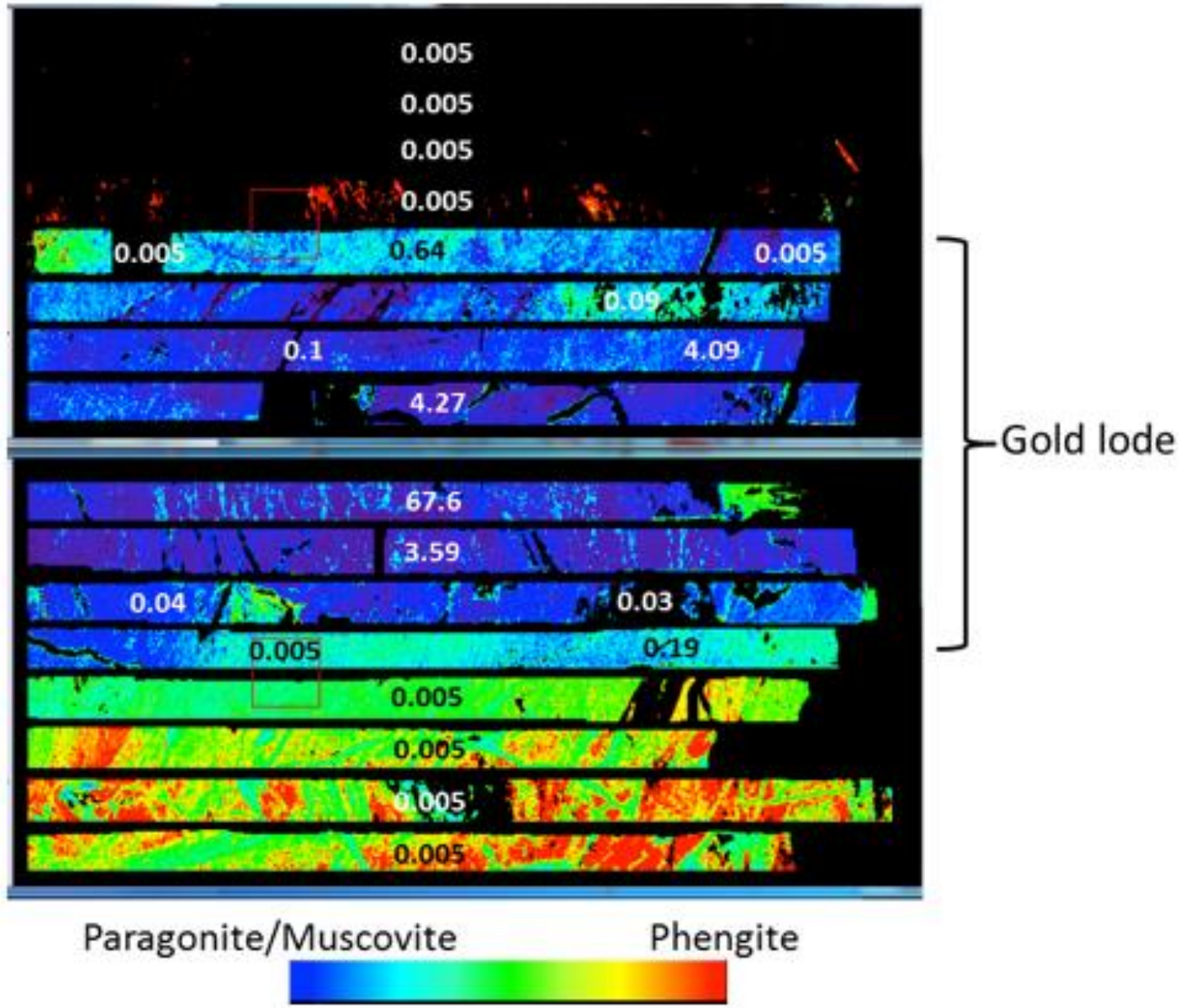
Colour stretch: 2190-2000nm

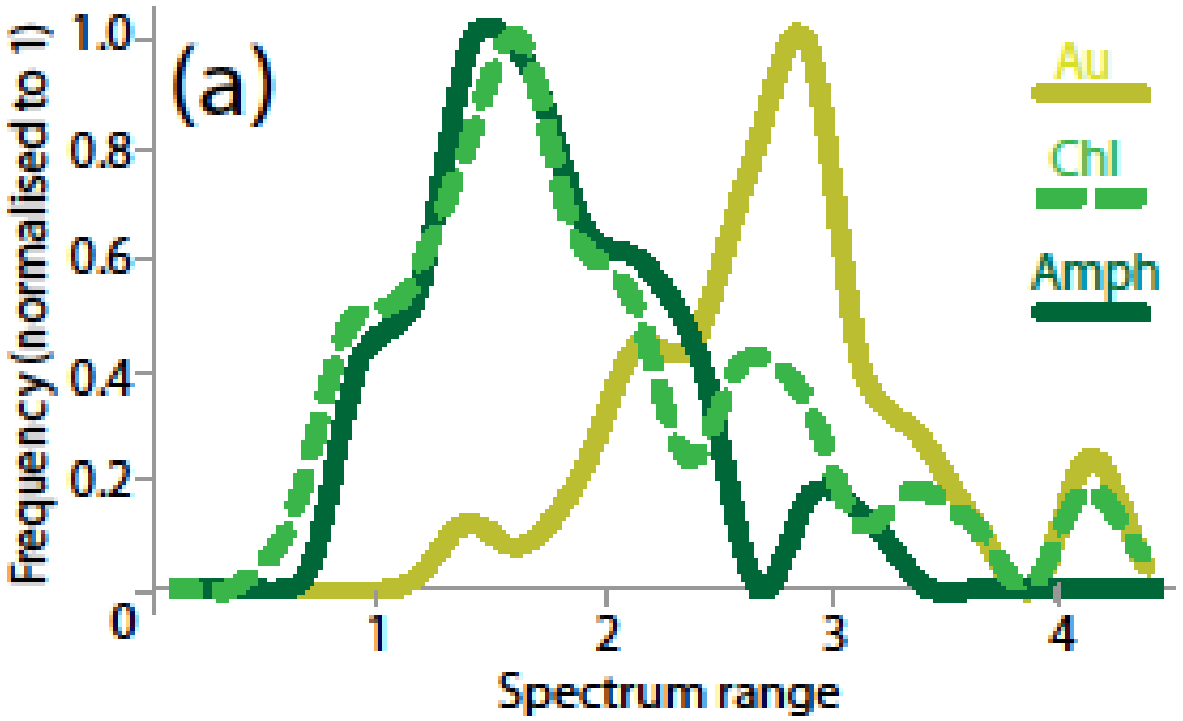
(Blue-red)

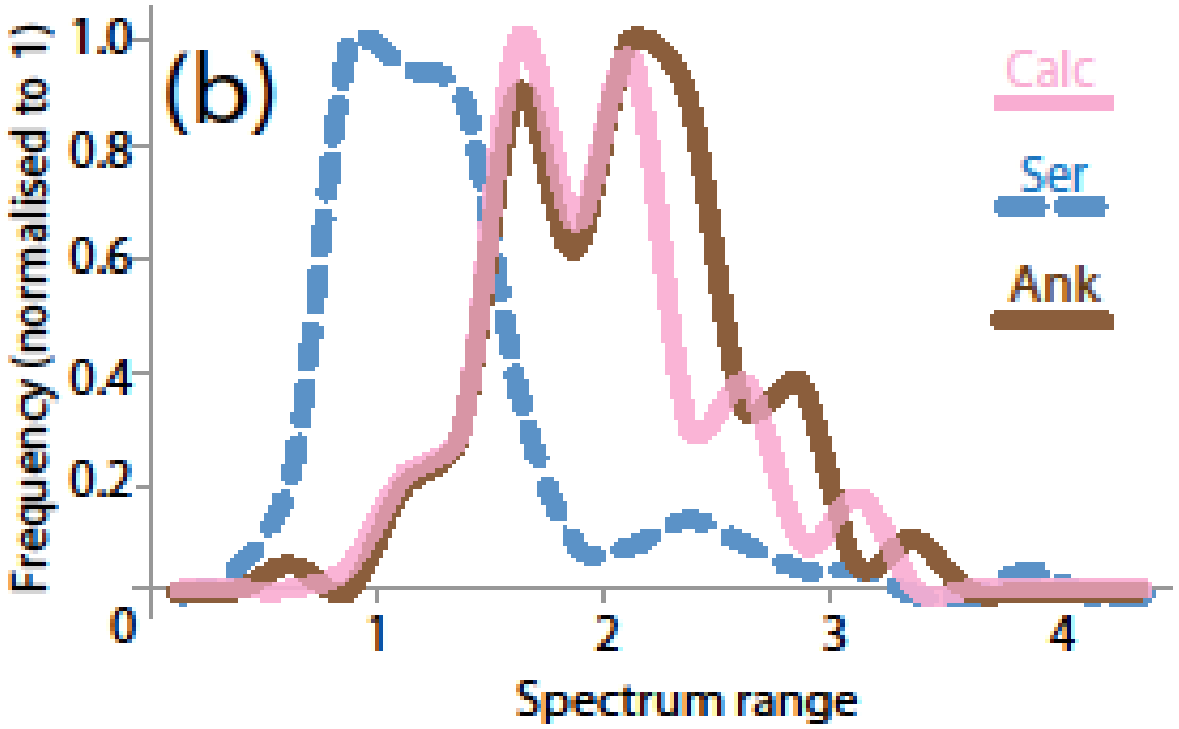


Near infra-red reflectance spectra of many kms of diamond drill core
→ detailed mineralogy + chemistry at mm / micron resolution

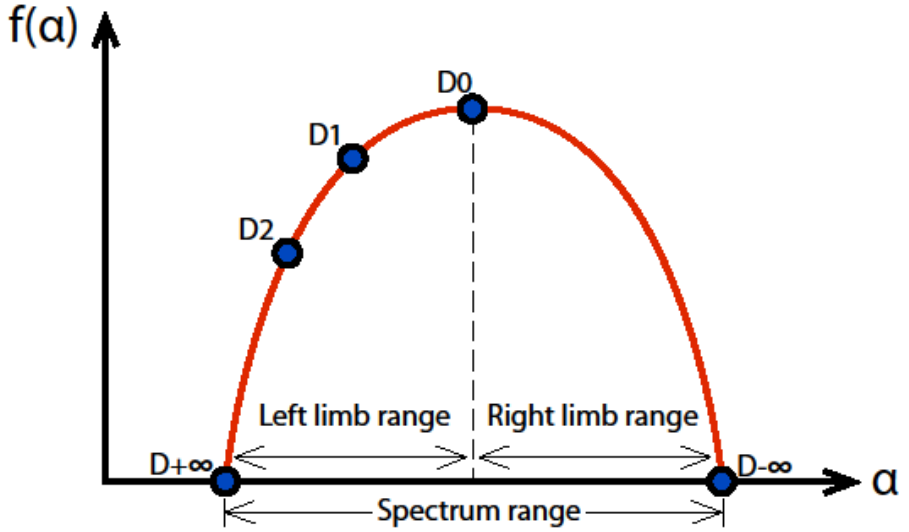
The colour → chemical composition of white micas from K-rich to Fe-rich

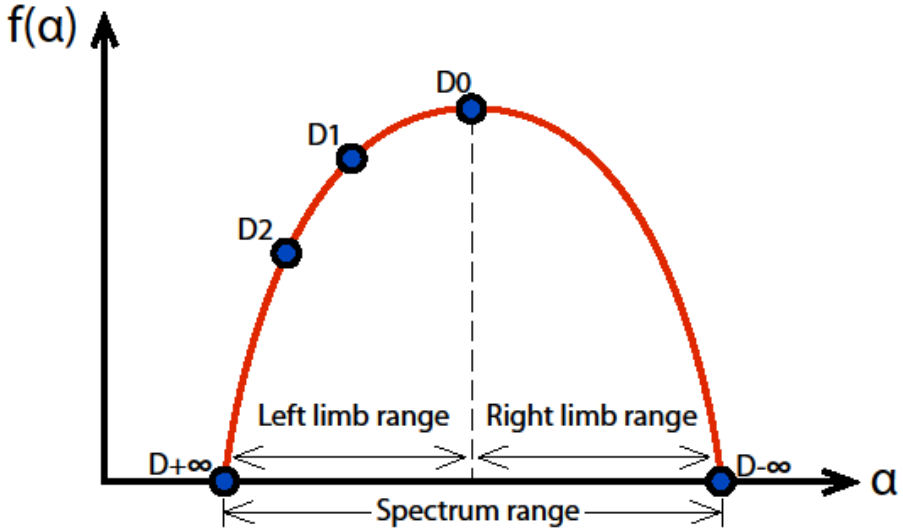

0.005
0.005
0.005
0.005
0.005
0.64
0.005
0.09
0.1
4.09
4.27
67.6
3.59
0.04
0.03
0.005
0.19
0.005
0.005
0.005
0.005
Gold lode
Paragonite/Muscovite
Phengite

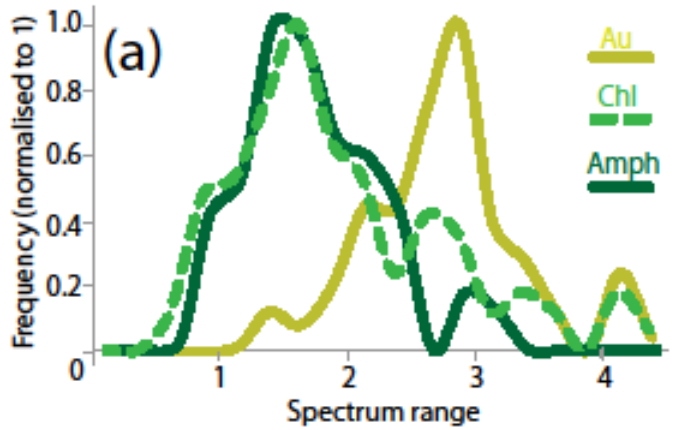


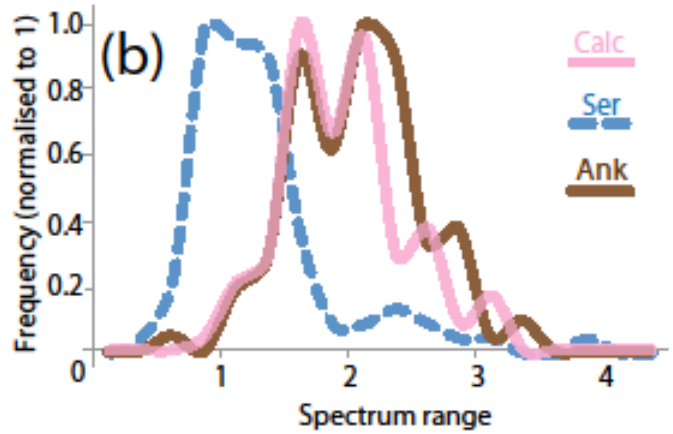


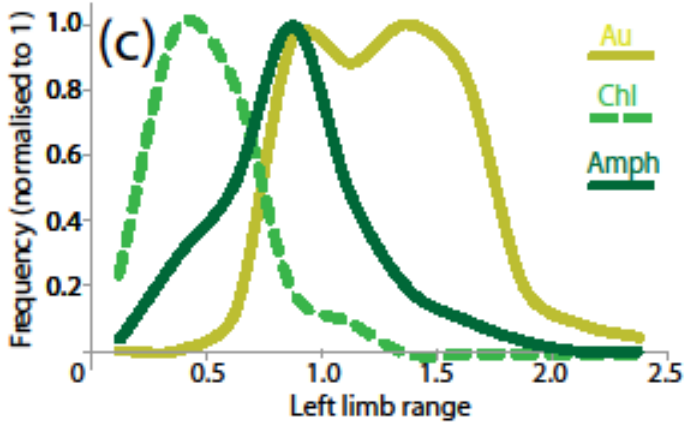
Remember how the width of the spectrum increases as the fluctuations increase

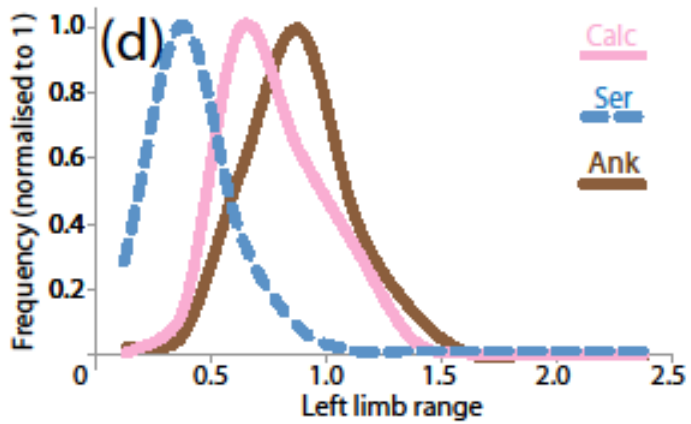


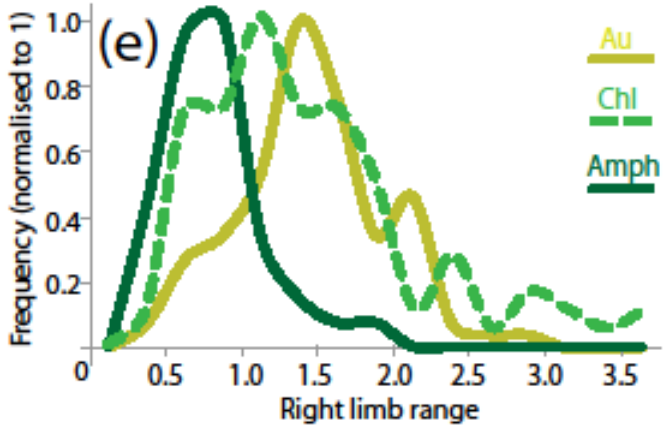

f(α)
D0
D1
D2
Left limb range
Right limb range
D+∞
D-∞
α
Spectrum range

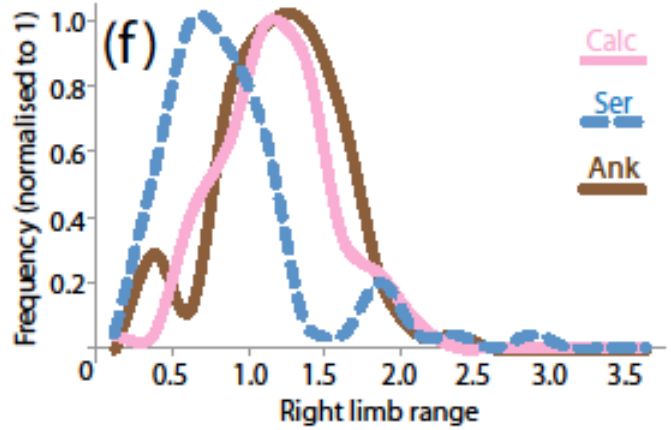

(a)
Au
Chl
Amph
Frequency (normalised to 1)
Spectrum range


(b)
Calc
Ser
Ank
Frequency (normalised to 1)
Spectrum range


(c)
Au
Chl
Amph
Frequency (normalised to 1)
Left limb range

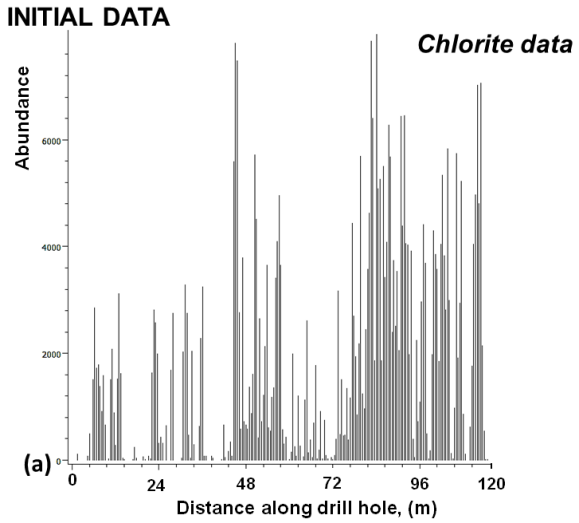

(d)
Calc
Ser
Ank
Frequency (normalised to 1)
Left limb range

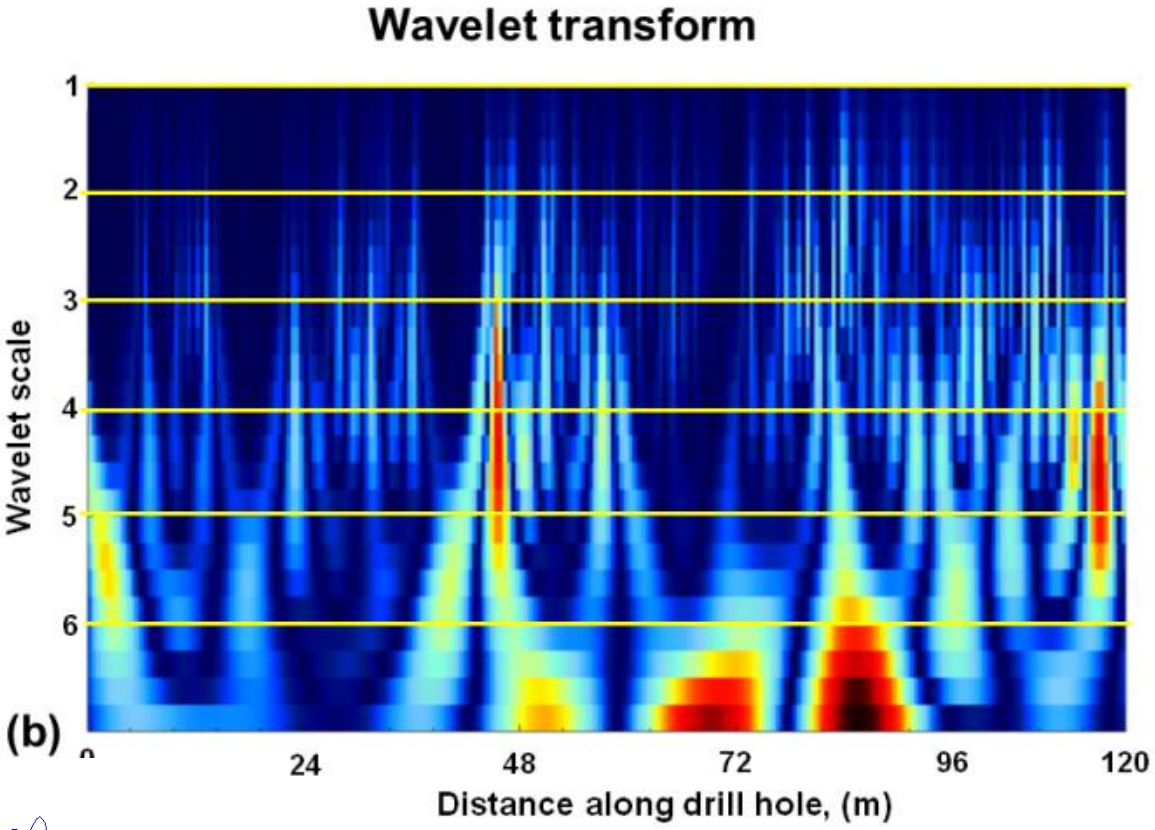

(e)
Au
Chl
Amph
Frequency (normalised to 1)
Right limb range

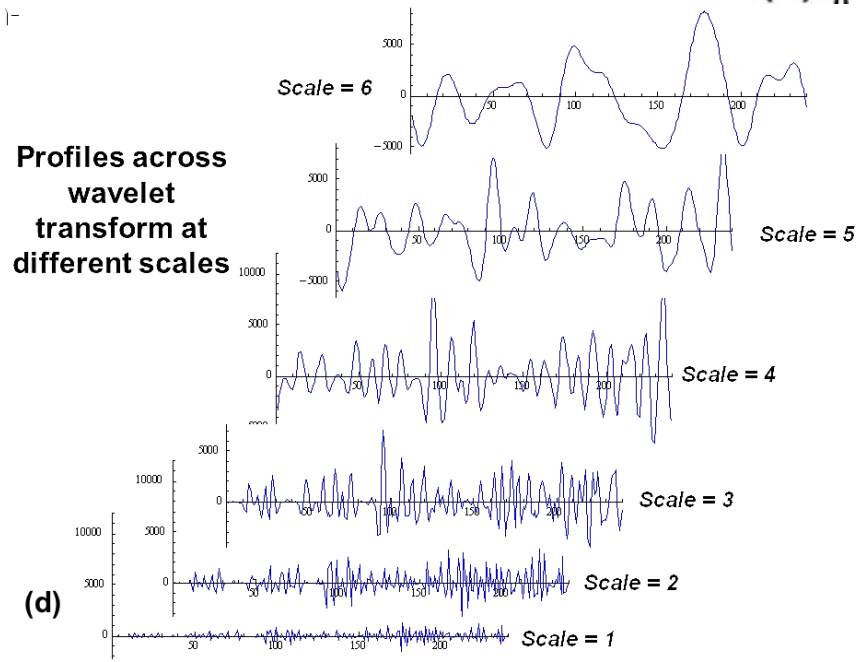

(f)
Calc
Ser
Ank
Frequency (normalised to 1)
Right limb range

Wavelet-based analysis of routinely acquired hyperspectral reflectance signals is demonstrated to be effective for quantifying the dynamical organization of down-hole primary mineralization, host rock alteration, and vein and breccia infill mineralogy.

Munro et al. 2017 Geol. Soc. Lond. Spec. Pub.

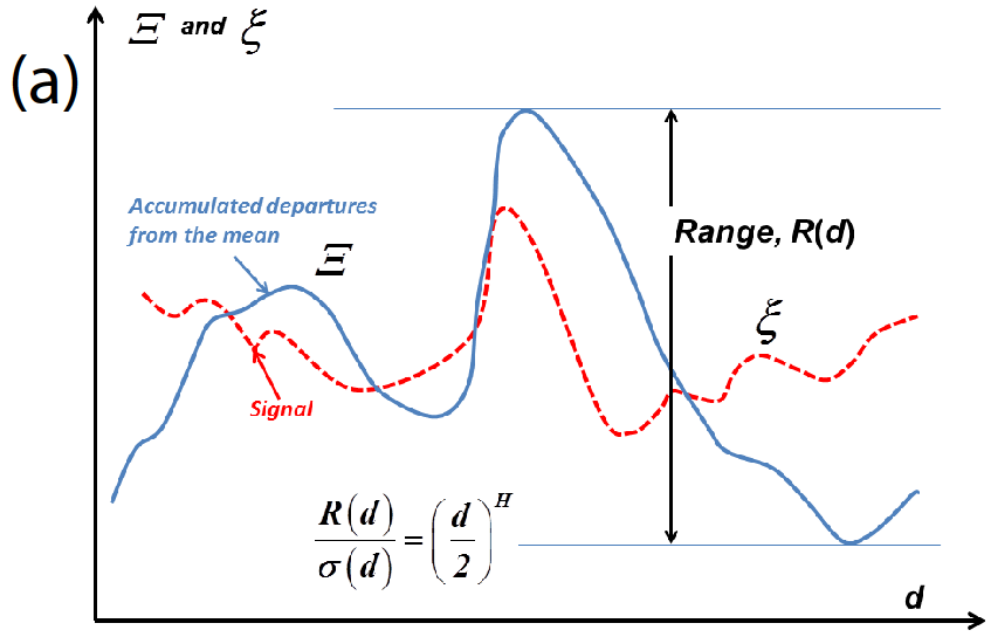


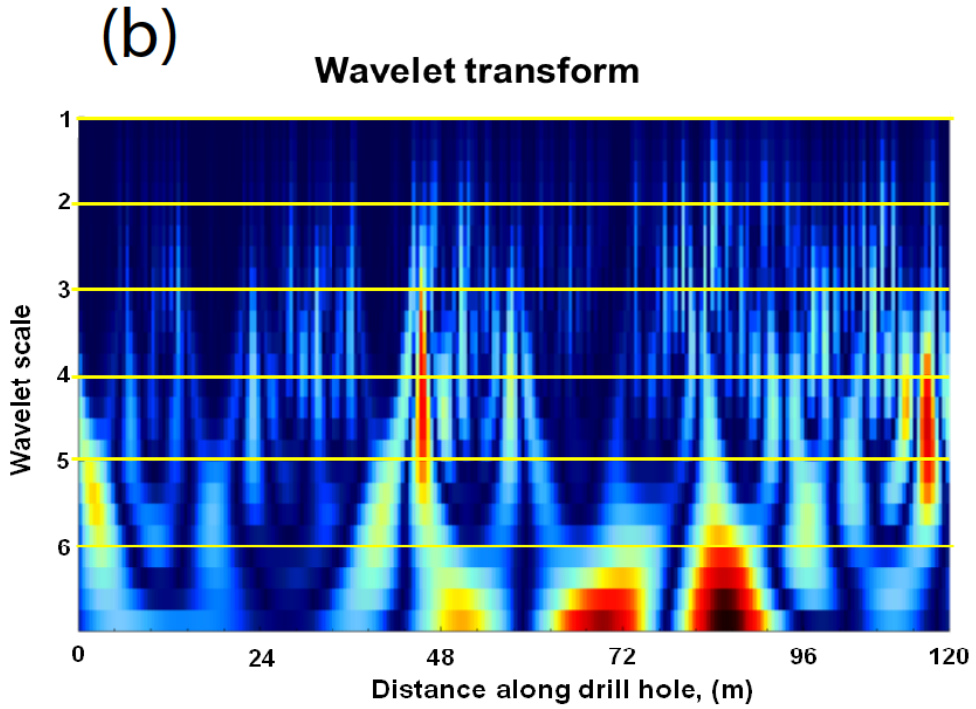


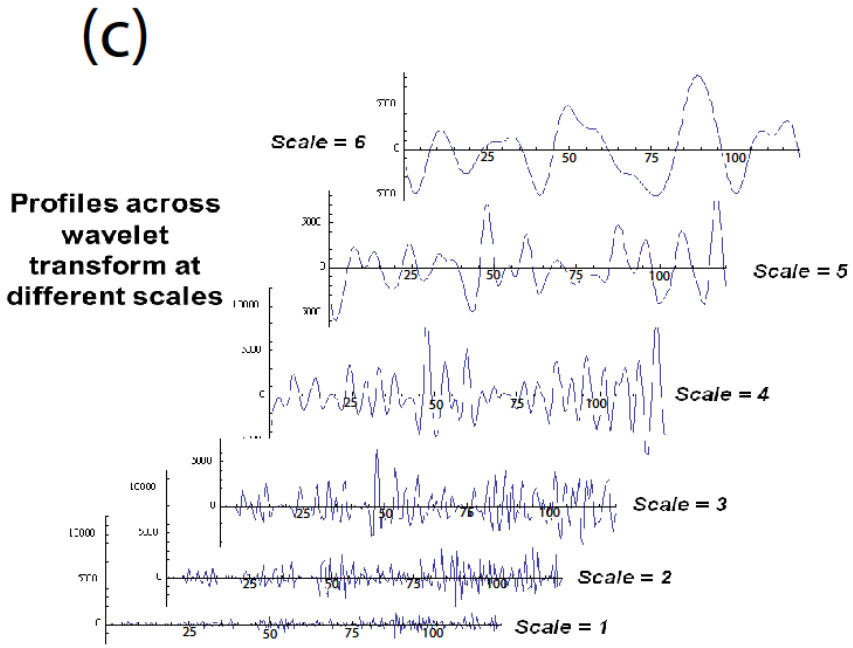


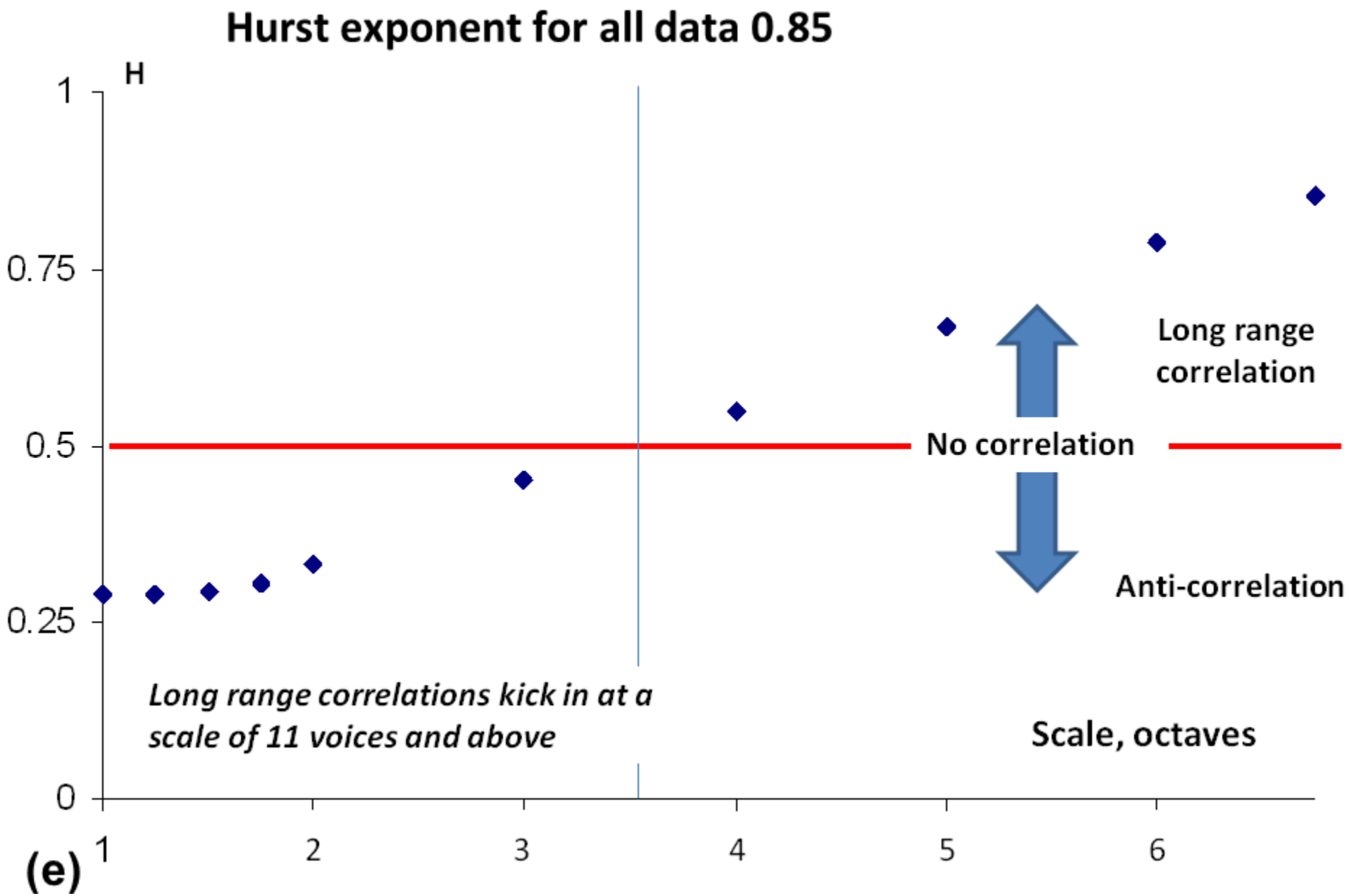
Derivation of signals at different scales

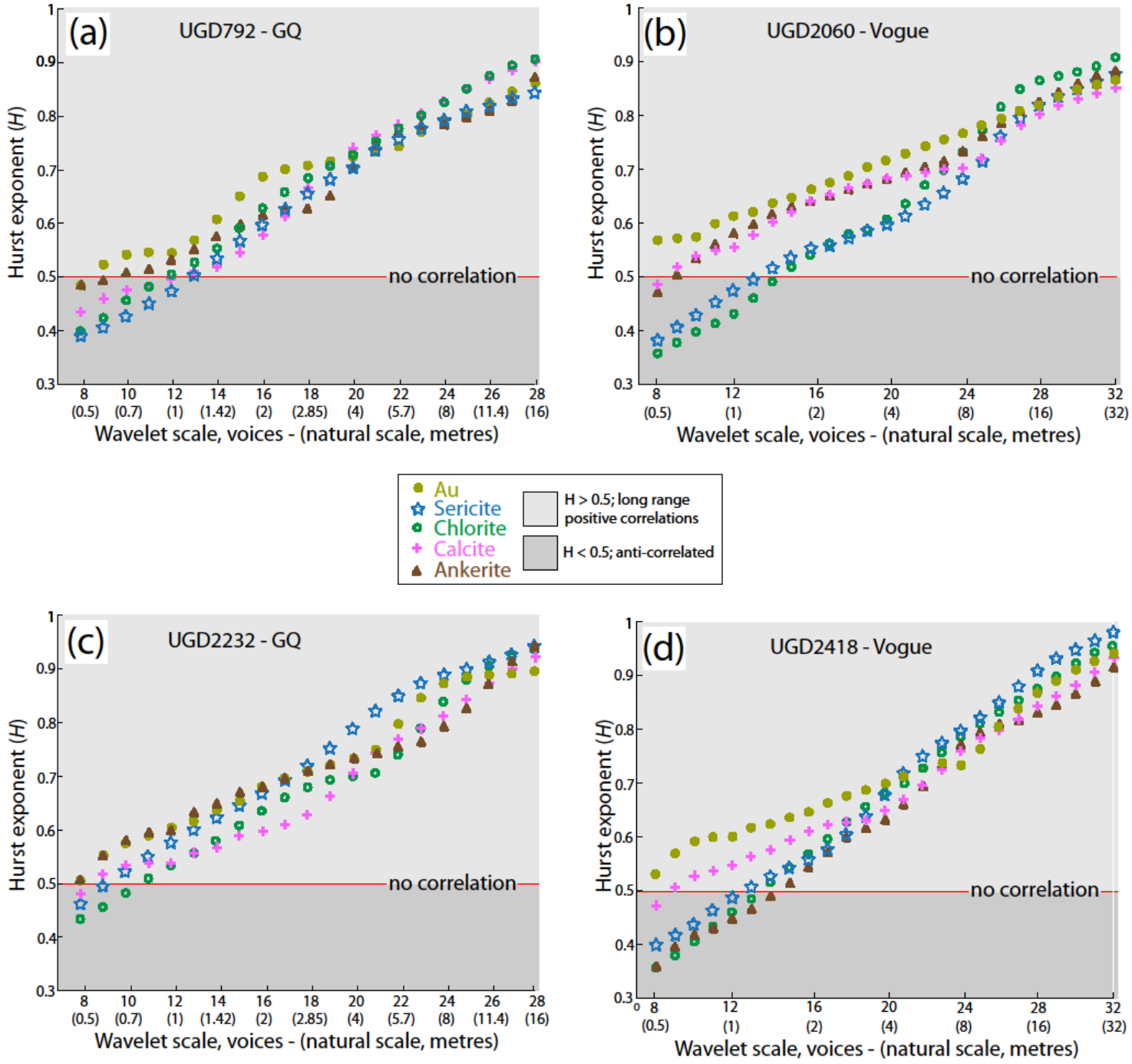
H Hurst exponent

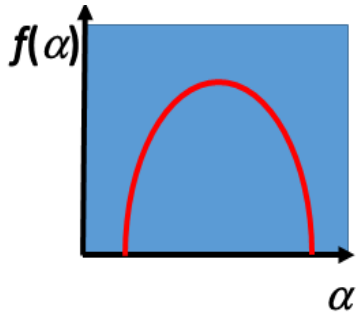







Hurst exponent for all data 0.85
H
1
0.75
0.5
0.25
0
1
2
3
4
5
6
Long range correlation
No correlation
Anti-correlation
Long range correlations kick in at a scale of 11 voices and above
Scale, octaves
(e)

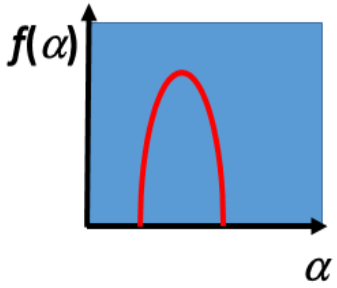

(a)
UGD792 - GQ
(b)
UGD2060 - Vogue
(c)
UGD2232 - GQ
(d)
UGD2418 - Vogue
Hurst exponent (H)
no correlation
Wavelet scale, voices - (natural scale, metres)
Au
Sericite
Chlorite
Calcite
Ankerite
H > 0.5; long range positive correlations
H < 0.5; anti-correlated



Large

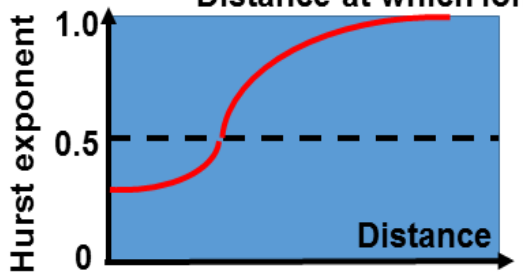
Highly endowed. Easy to mine	Highly endowed. Difficult to mine
Poorly endowed. Easy to mine	Poorly endowed. Difficult to mine

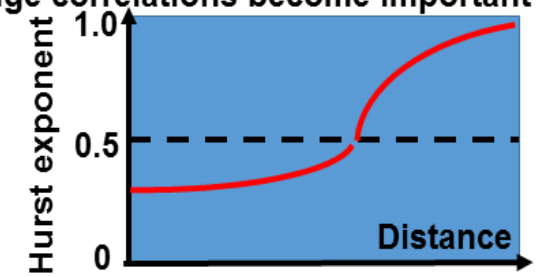
$(D_{-\infty} - D_{+\infty})$



Small

Distance at which long range correlations become important

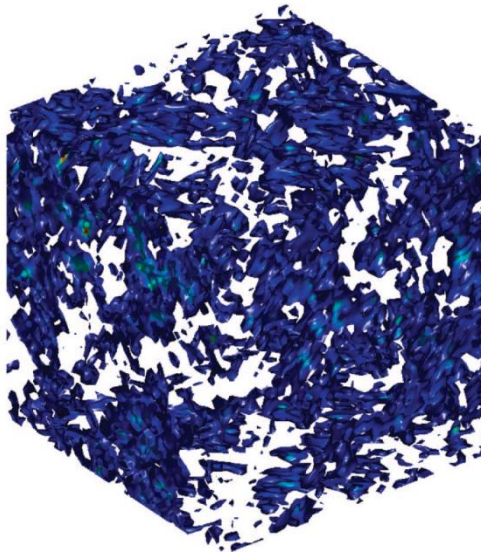




A classification of ore bodies on the basis of width of the singularity spectrum and whether long range correlations appear at small or large spatial scales relative to the scale of day to day mining operations.

Hydrothermal mineralising systems are nonlinear dynamical systems and need to be studied using the tools of nonlinear dynamics

This involves looking for long range order in the system and whether the overall patterns of mineralisation and alteration are multifractal



A fractal system. The elements of the system do not fill space

A multifractal system. A set of intertwined fractals each of which fill space to varying degrees.