Structural Geology & Resources 2022
16 October 2022

Mineral systems as chemical reactors with no mathematics

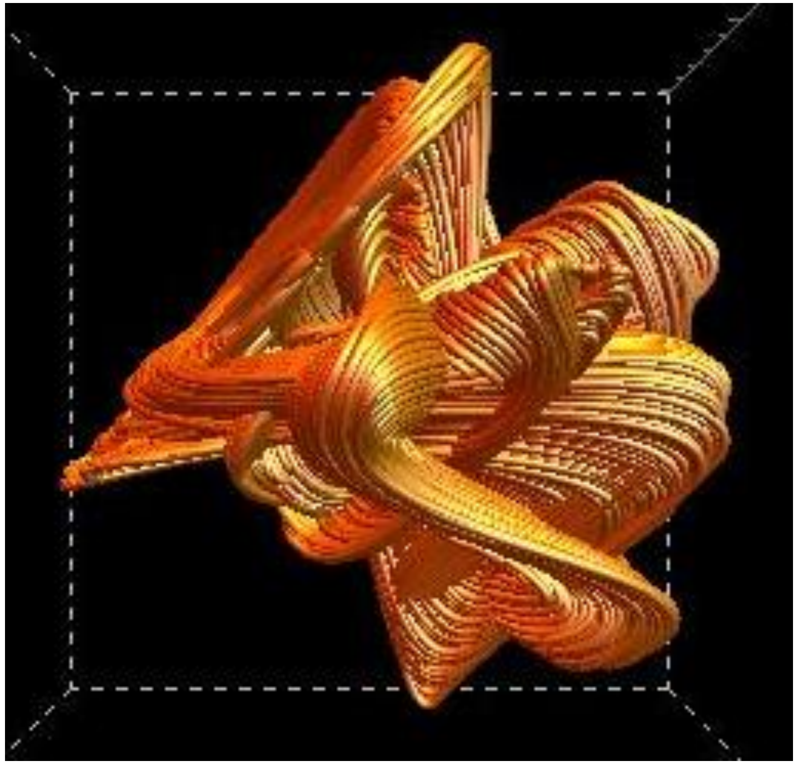
Bruce Hobbs and Alison Ord

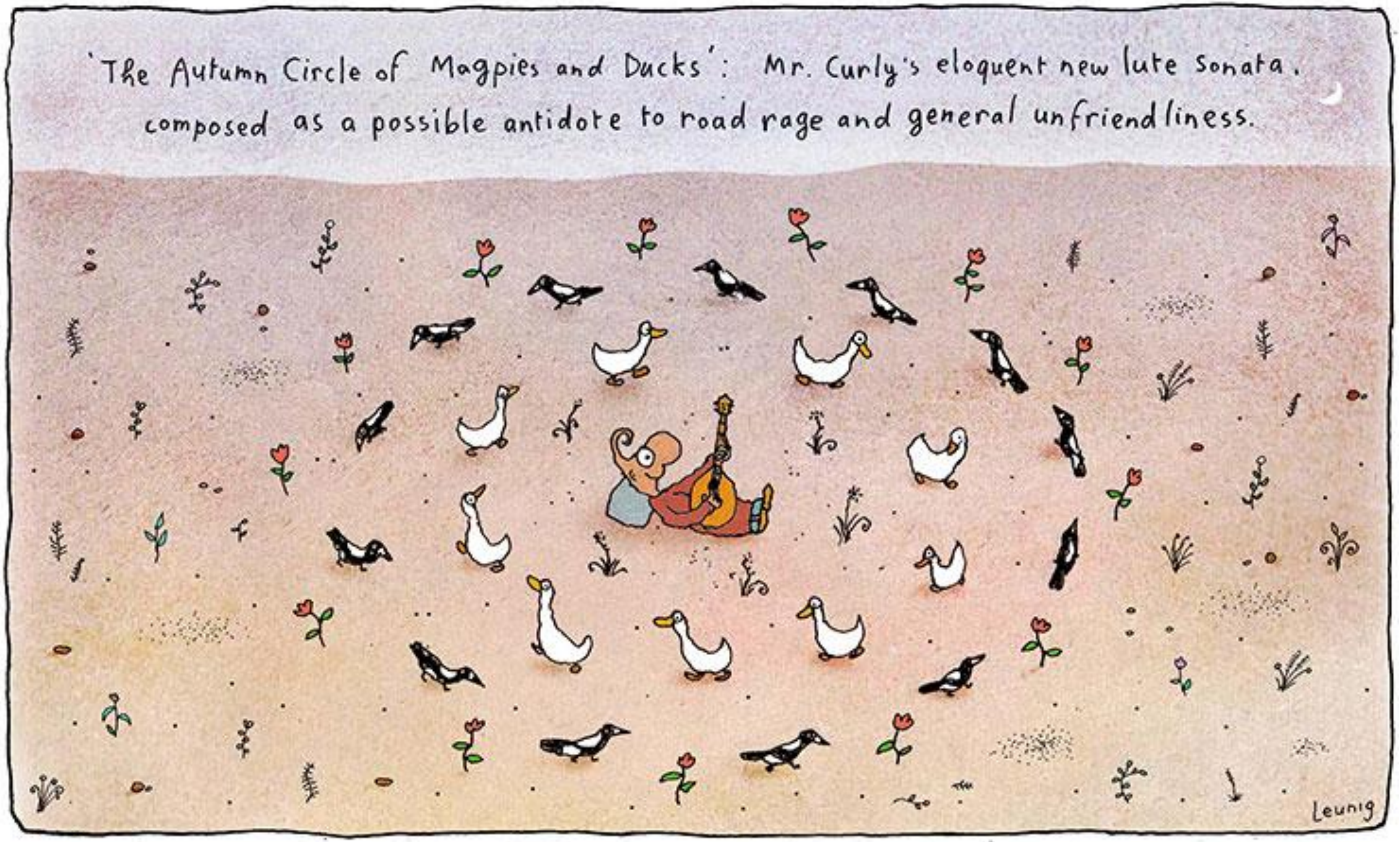
Session 4. 10.45 – 12.30

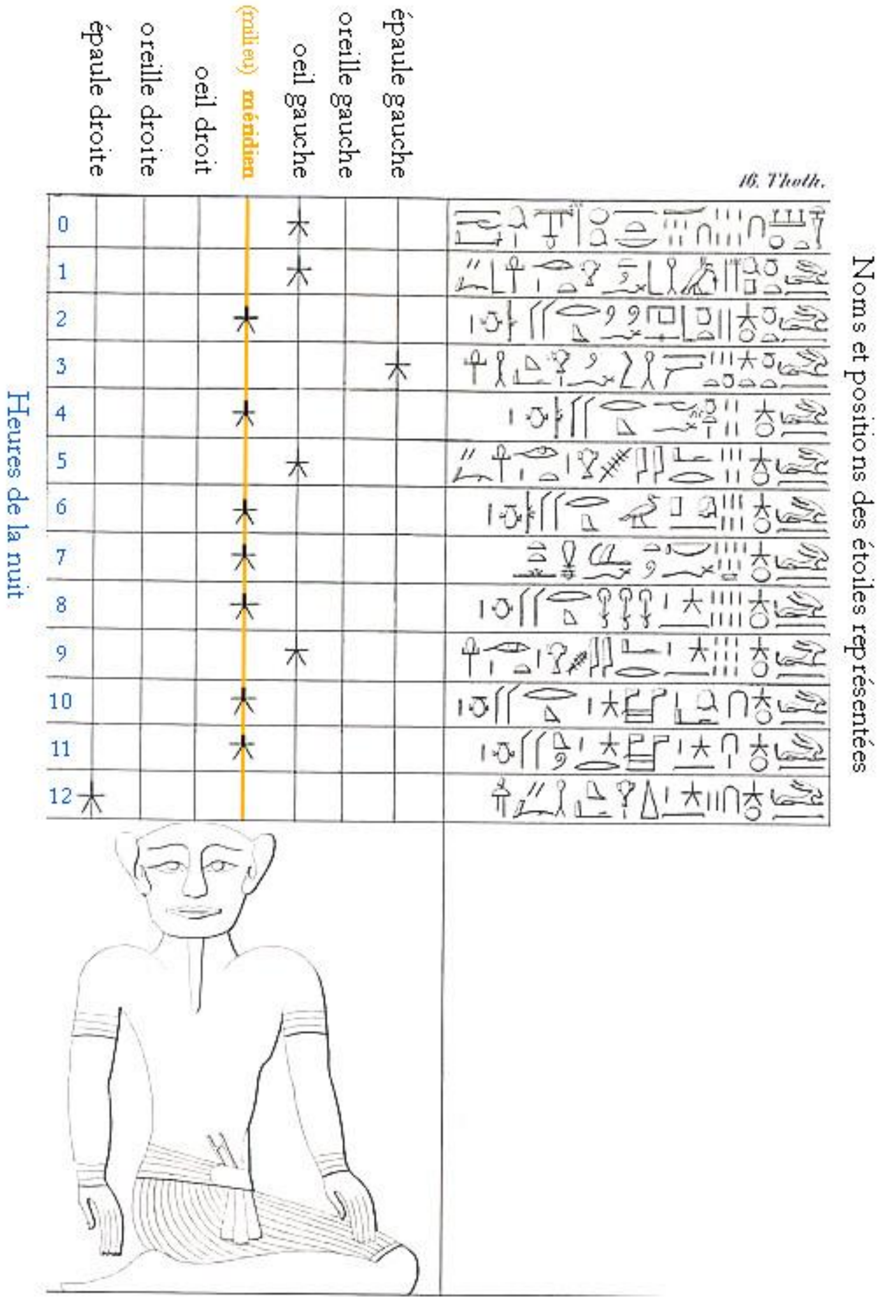
The nonlinear toolbox: What do we do with all these data?

- Wavelets and multifractals. Long range correlations.
- **Recurrence.**
- Probability distributions

Fingerprint for a mineralising system



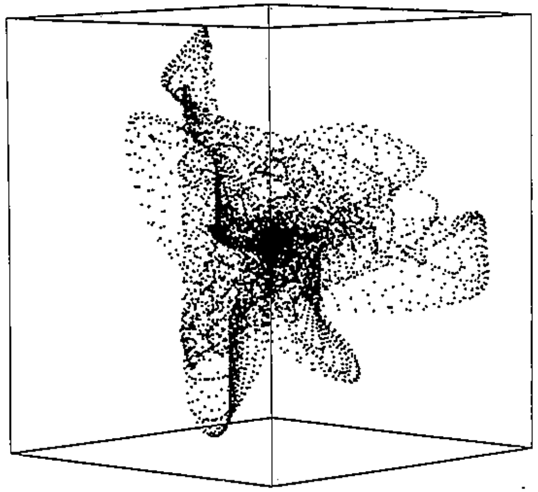

'The Autumn Circle of Magpies and Ducks': Mr. Curly's eloquent new lute sonata.
composed as a possible antidote to road rage and general unfriendliness.
Leunig



The recurrence of states, states being again arbitrarily close after some time of divergence, is a fundamental property of deterministic dynamical systems and is typical for nonlinear or chaotic systems.

The recurrence of states in nature has been known for a very long time. More recent work is that of Henri Poincare in 1890.

Eckmann et al. (1987) introduced recurrence plots, which provide a way to visualize the periodic nature of a trajectory through a phase space.



Often, the phase space does not have a low enough dimension (two or three) to be pictured, since higher-dimensional phase spaces can only be visualized by projection into the two or three-dimensional sub-spaces.

However, making a recurrence plot enables us to investigate certain aspects of the *m*-dimensional phase space trajectory through a two-dimensional representation.

A recurrence is a time the trajectory returns to a location it has visited before.

For any given moment, a recurrence plot (RP) is a plot which shows the times at which a phase space trajectory visits roughly the same area in the phase space.

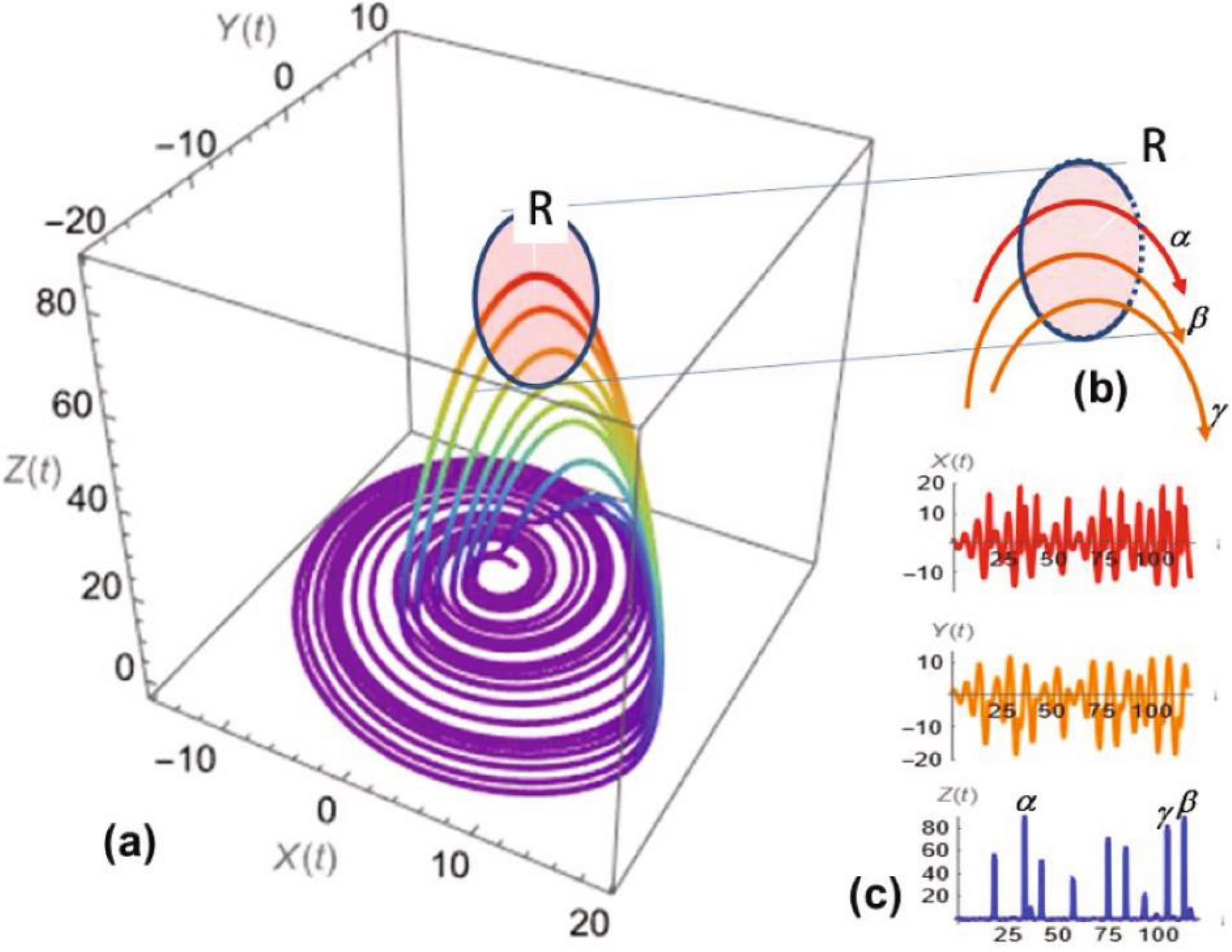
The recurrence plot depicts the collection of pairs of times at which the trajectory is at the same place.

Construct a graph with time i on the horizontal axis and time j on the vertical axis.

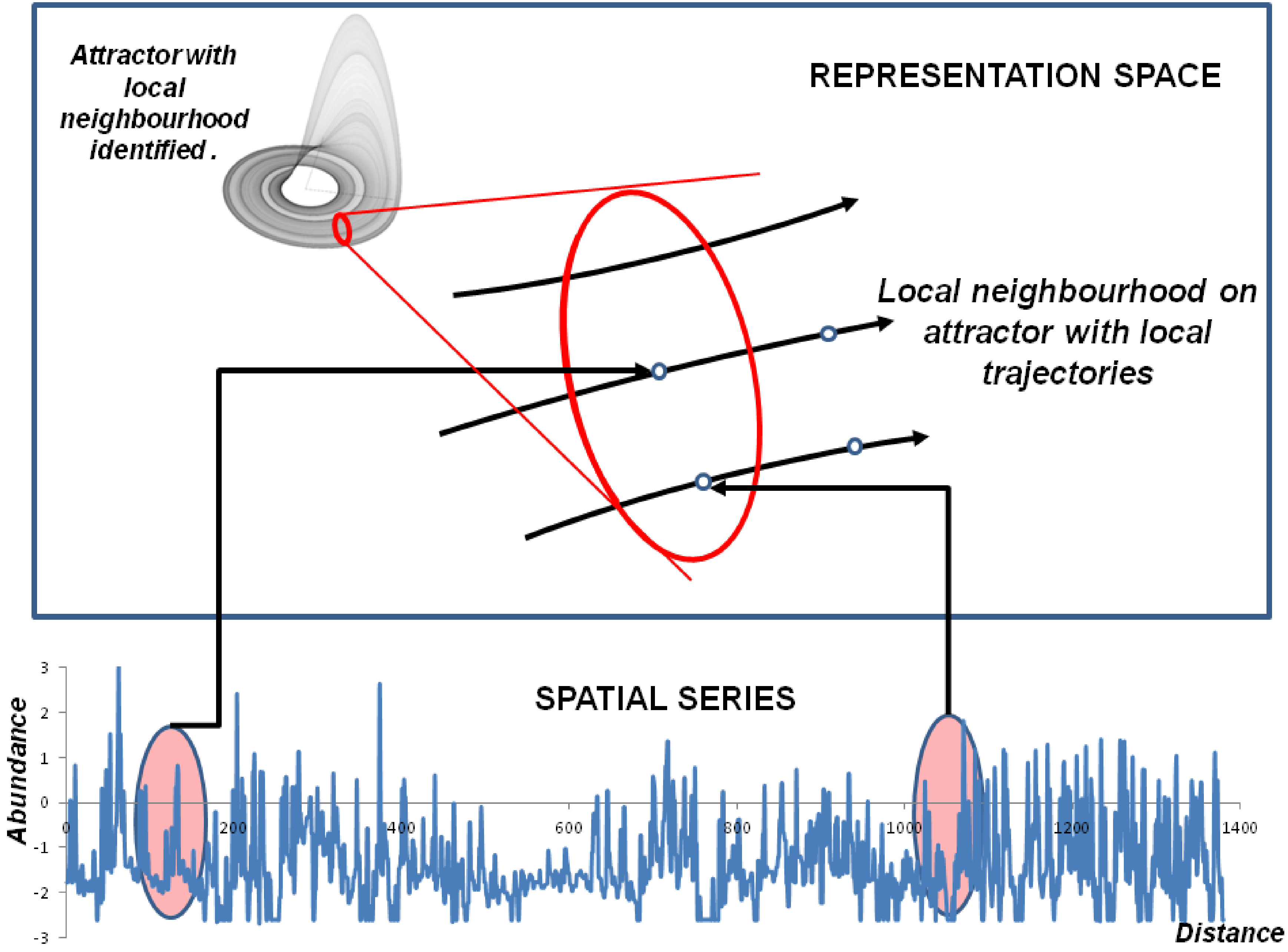
Plot $\vec{x}$ the trajectory through phase space for time i and for time j.

Any point then represents a trajectory visited at time and also time j.

$$\vec{x}(i) \approx \vec{x}(j)$$



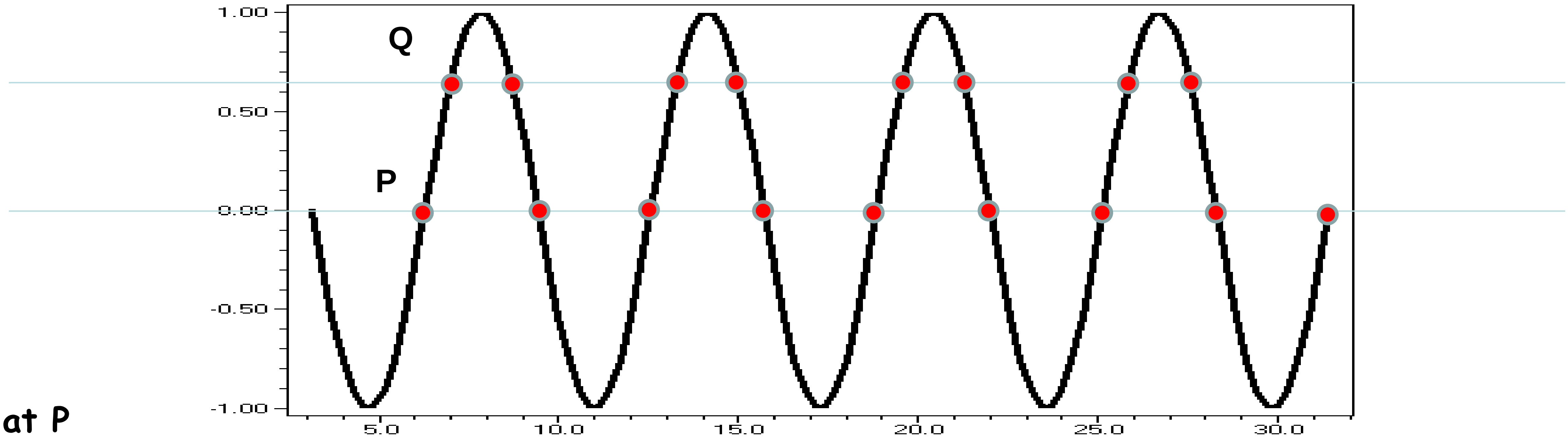
The concept of recurrence.
(a) Shows the attractor for the Rossler system (Sprott, 2003). This represents the reaction between three chemical compounds, *X*, *Y* and *Z*. Since the attractor for a non-random system does not occupy all of phase space the system repeatedly passes close to a former state on the attractor as it evolves. Here the trajectories of the system pass through states within the red ellipse, R.
In (b) some of these trajectories are shown and labelled *α*, *β*, γ. (c). The evolution of the concentrations of the compounds *X*, *Y* and *Z*. The recurrence is marked by *α*, *β*, γ in the evolution of *Z*.
The smaller peaks in (c, lower) are outside the tolerance set by the ellipse in (a).
One way of quantifying how closely a given state is repeated as the system evolves is to use recurrence plots.



Attractor with local neighbourhood identified.
REPRESENTATION SPACE
Local neighbourhood on attractor with local trajectories
SPATIAL SERIES
Abundance
Distance
3
2
1
0
-1
-2
-3
0
200
400
600
800
1000
1200
1400

How do we measure recurrence?

Let us look first at a sine wave



Start at P
Clearly there is a recurrence (a place with the same value) every half wavelength

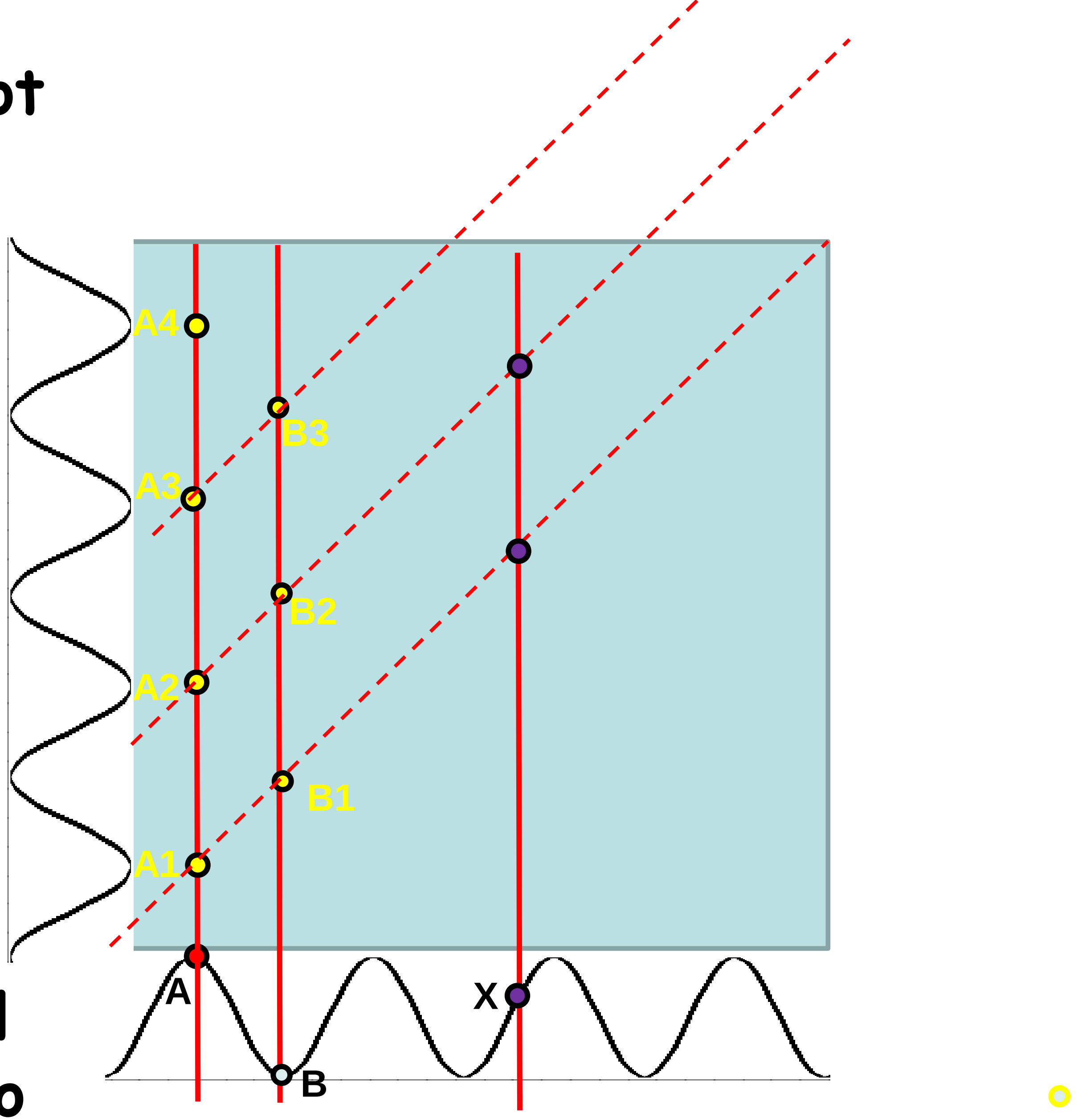
Start at Q
Clearly there is a recurrence which is regular but not every half wavelength

Construction of a recurrence plot for a sine wave

Take a point A on the signal and plot all points that recur. This gives us A1, A2, A3, ……

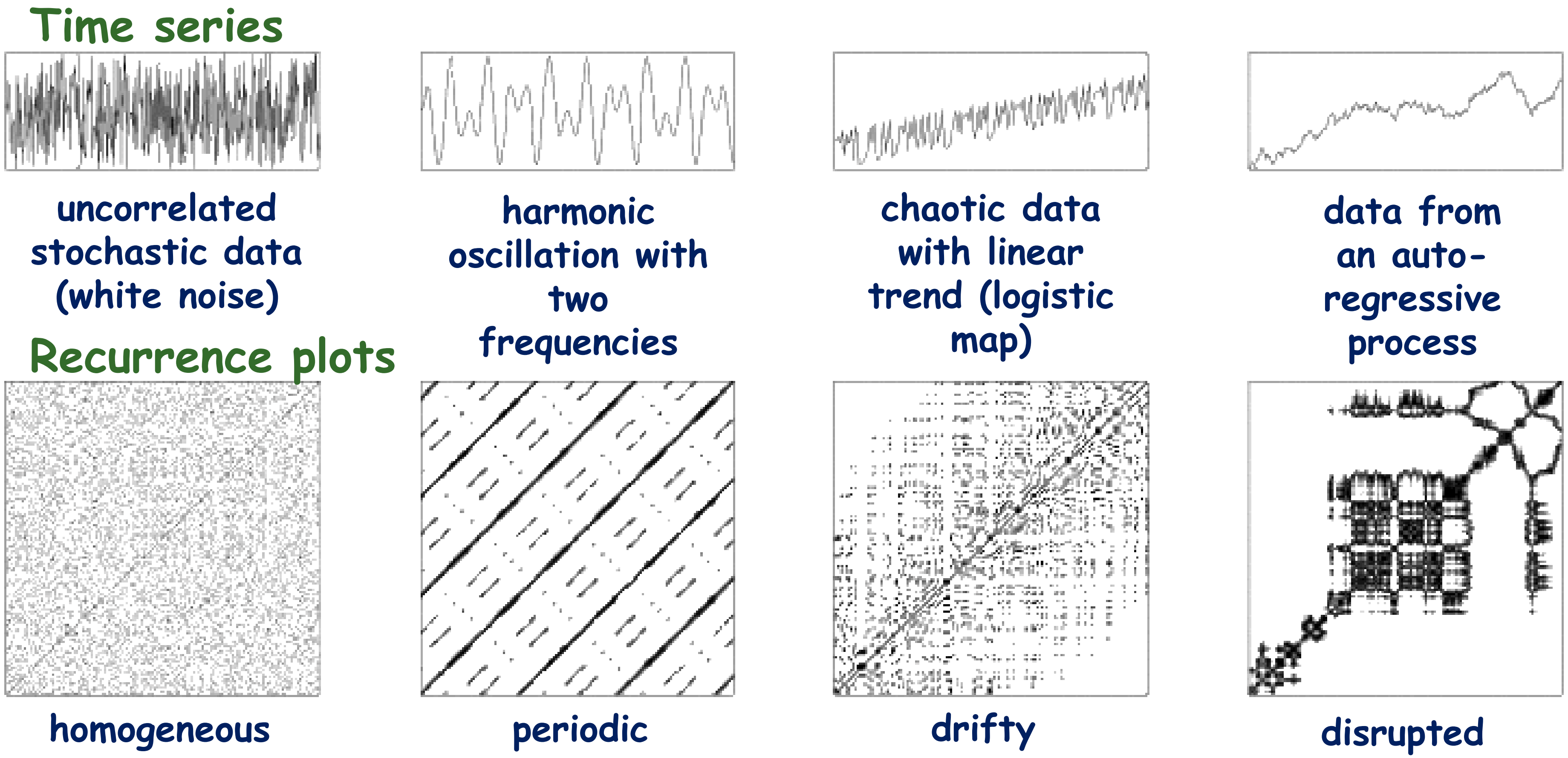
Do the same for another point, B giving B1, B2, B3, ………

Repeat for any other point X.



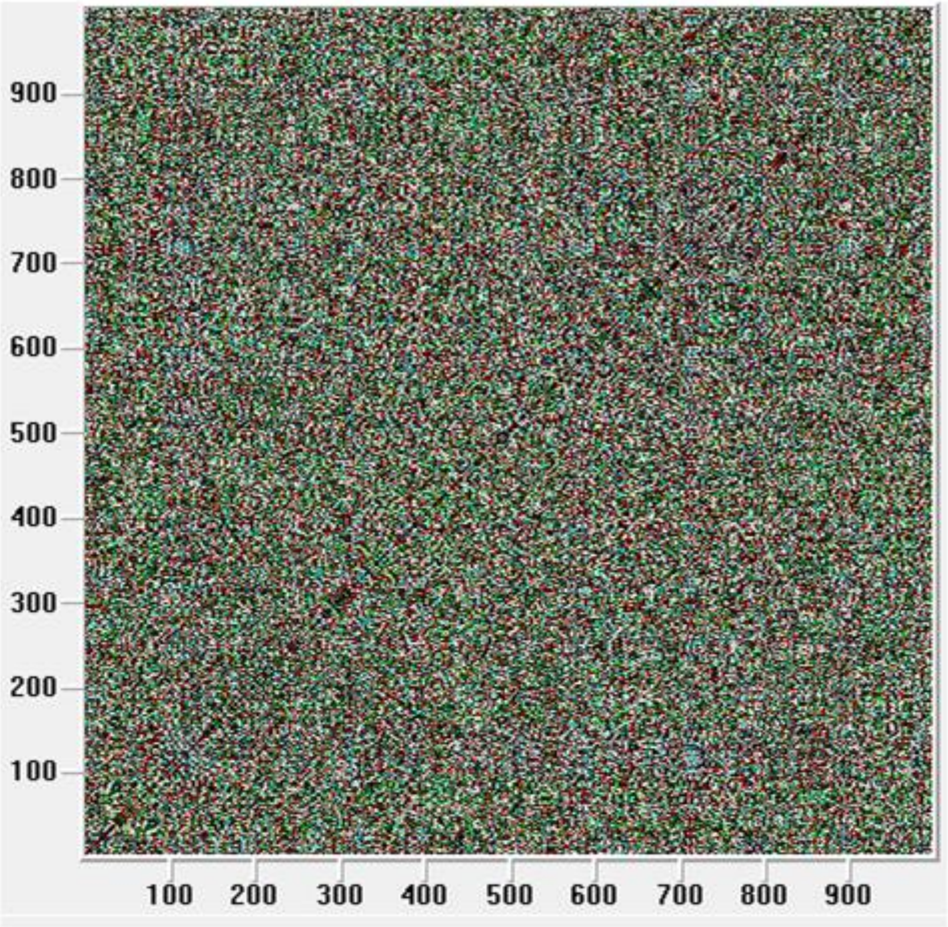
The result is a series of diagonal lines with vertical spacing equal to the period of the signal

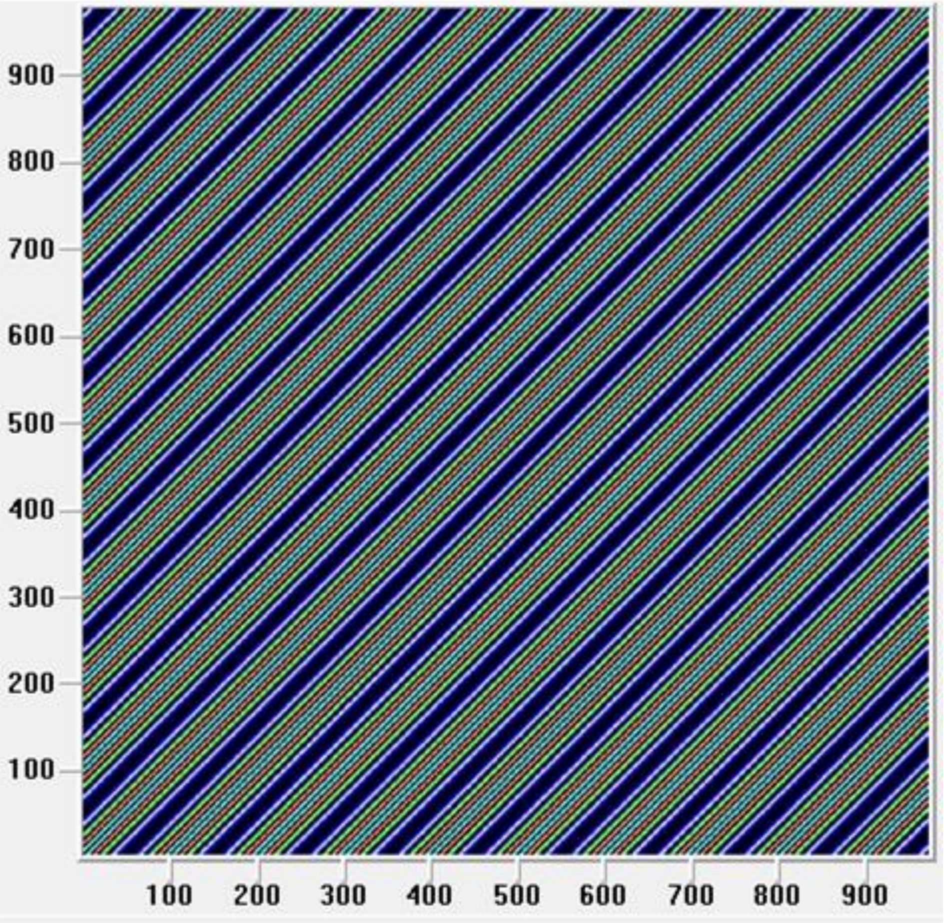
Typical examples of recurrence plots



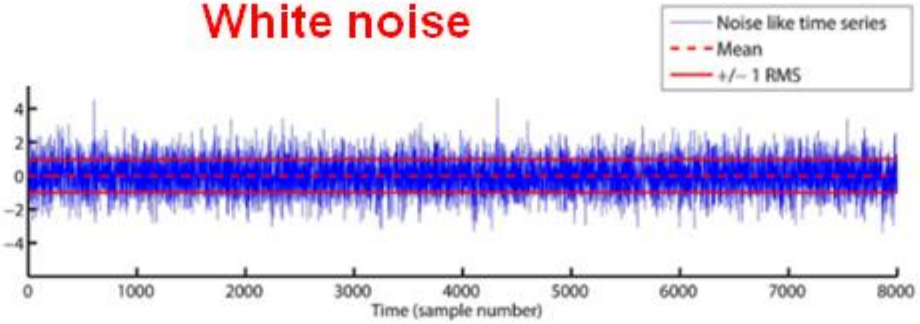
Transitions or nonlinear parameters of the system may be determined from the structures of the RPs.

Some examples of recurrence plots

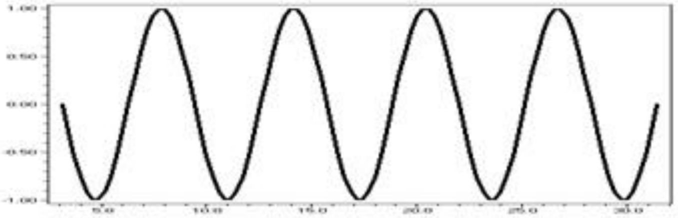


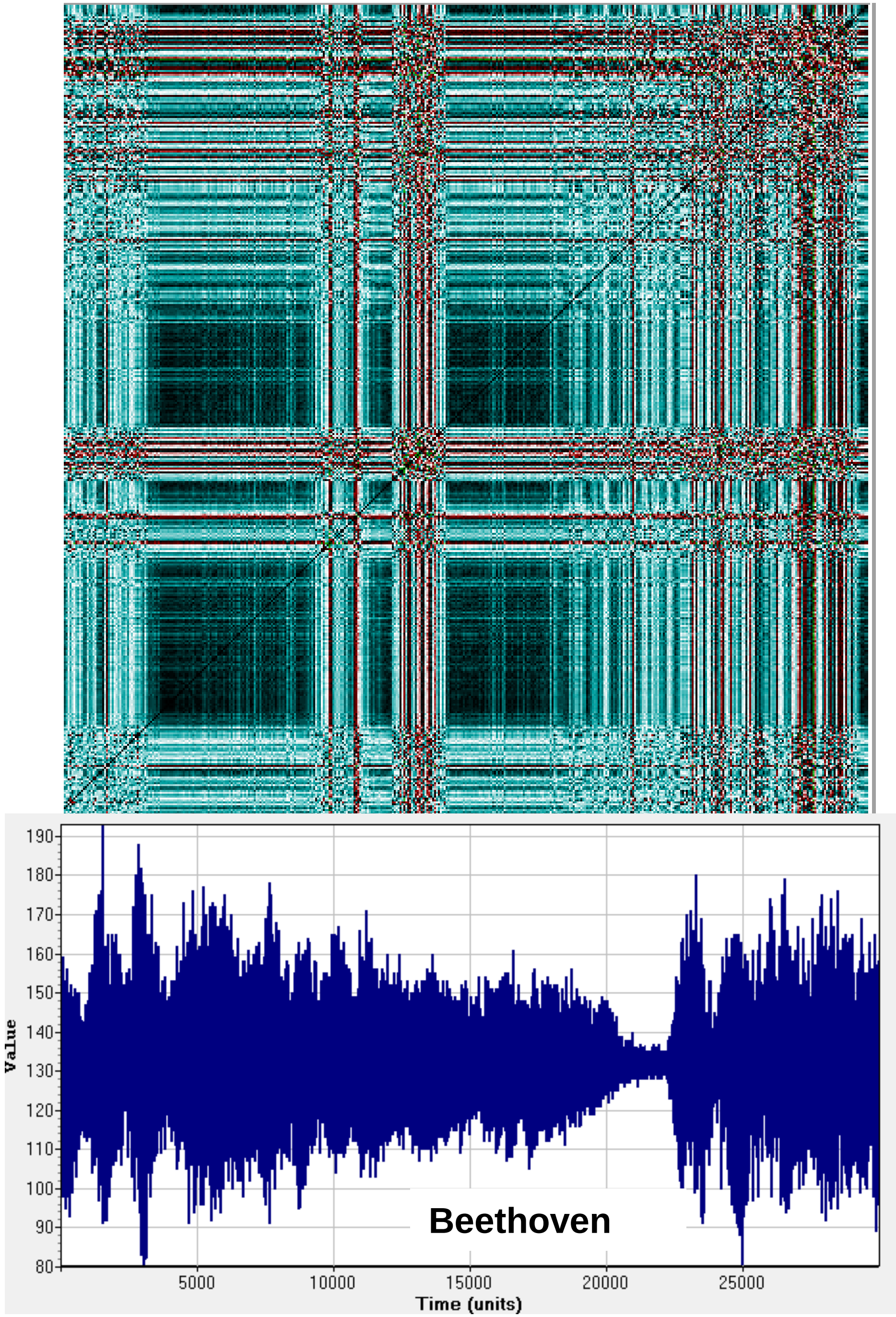


White noise

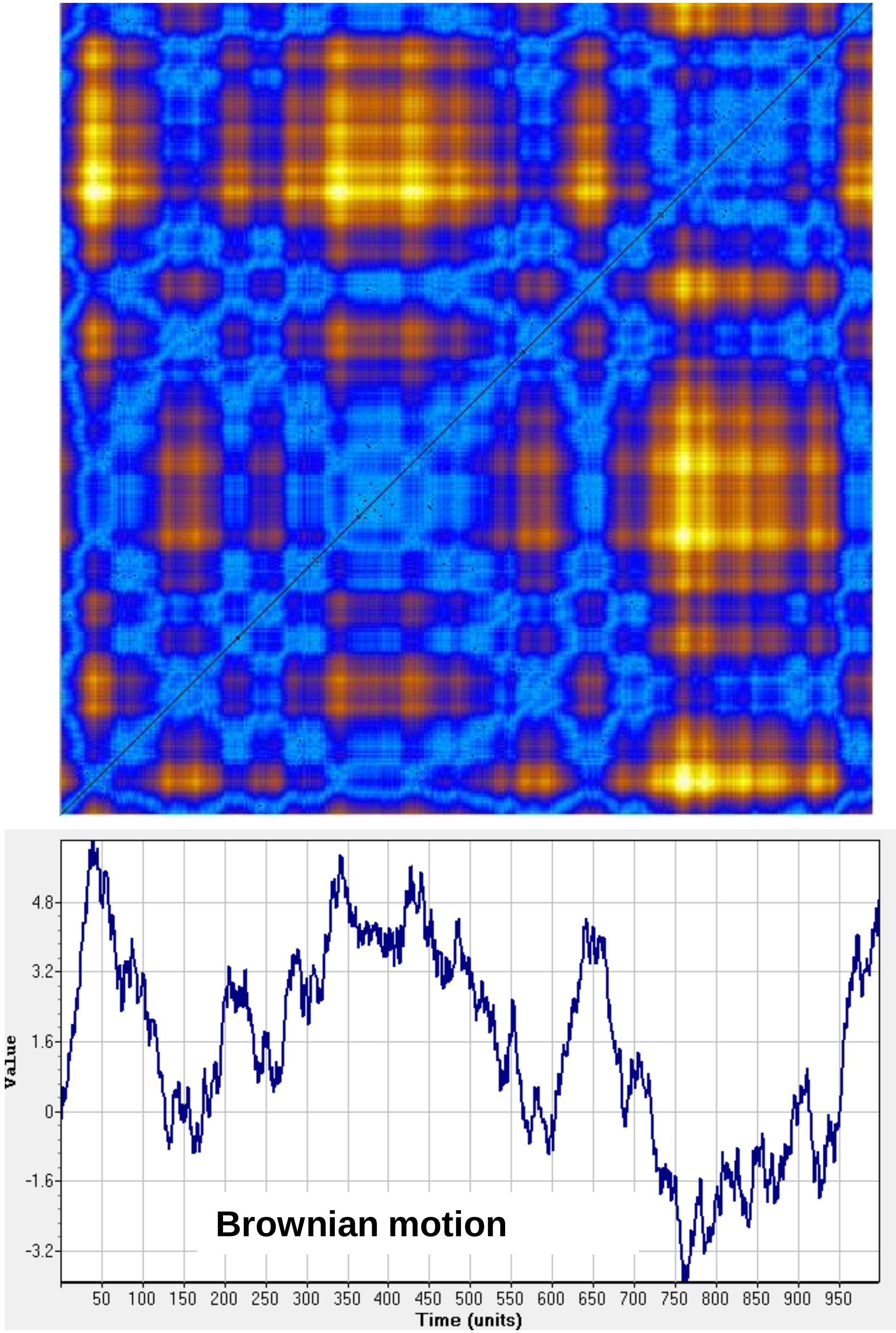


Sine wave





Beethoven
Value
190
180
170
160
150
140
130
120
110
100
90
80
5000
10000
15000
20000
25000
Time (units)



Brownian motion
Value
4.8
3.2
1.6
0
-1.6
-3.2
50
100
150
200
250
300
350
400
450
500
550
600
650
700
750
800
850
900
950
Time (units)

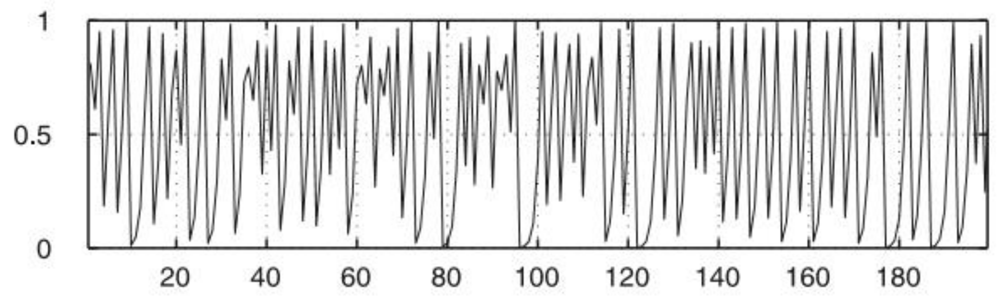
Logistic function

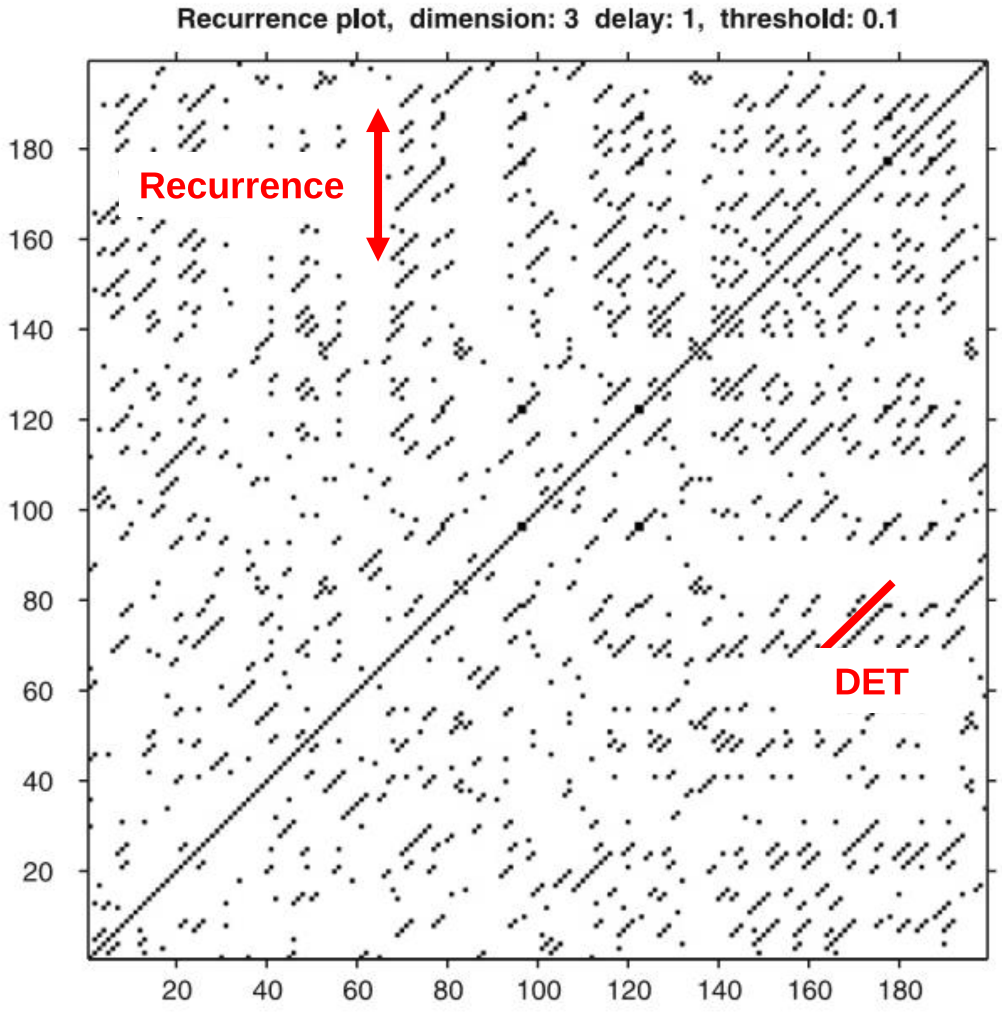
$$x_{n+1} = \lambda x_n \left(1 - x_n\right)$$

Simple deterministic relation to express the competition between two processes

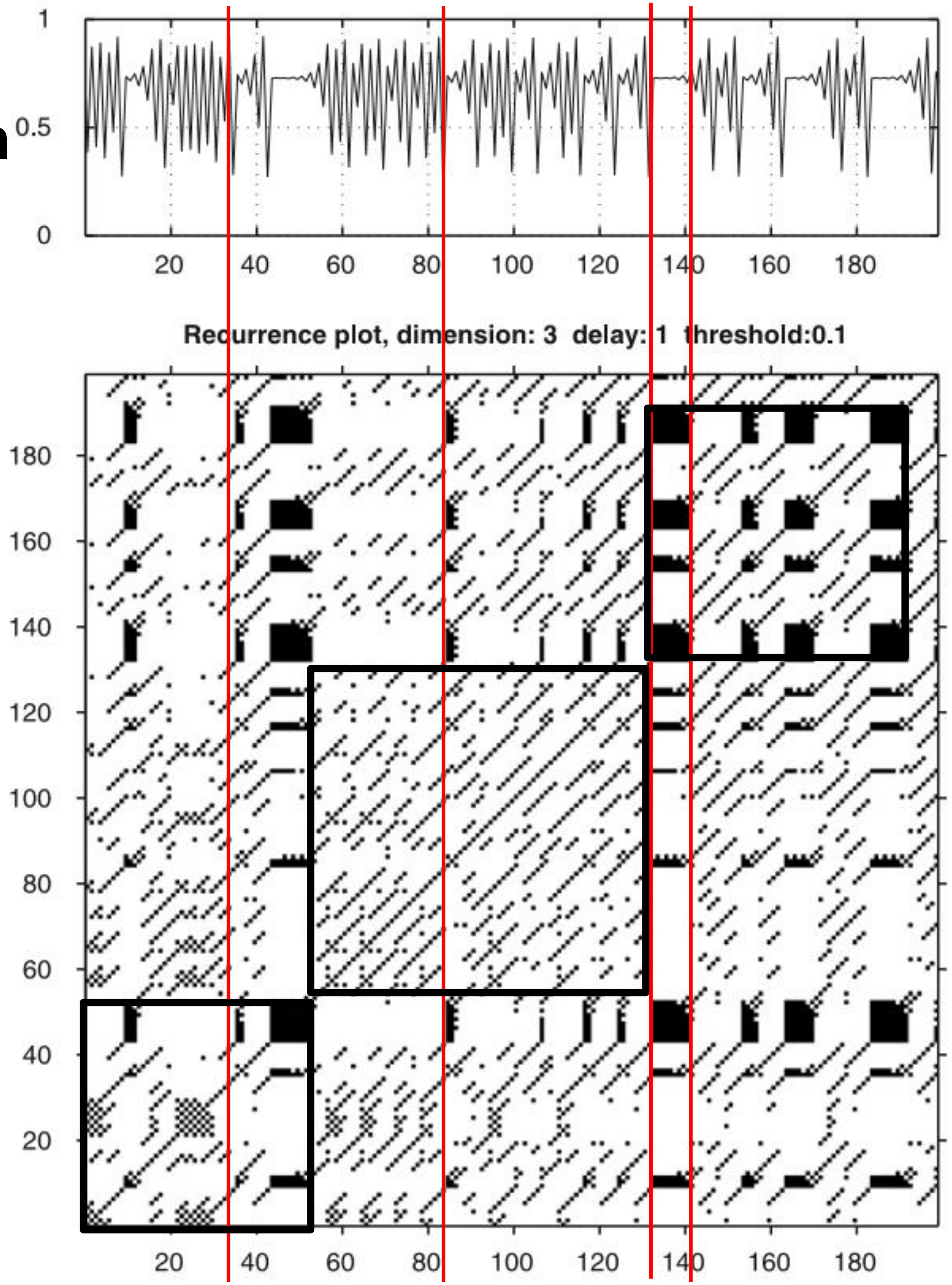
The frequency distribution of the lengths of the diagonal lines give a quantitative measure of determinism or of the degree of predictability in the signal. This is the *determinism*, DET.

The frequency distribution of vertical distances between diagonal lines gives the recurrence histogram



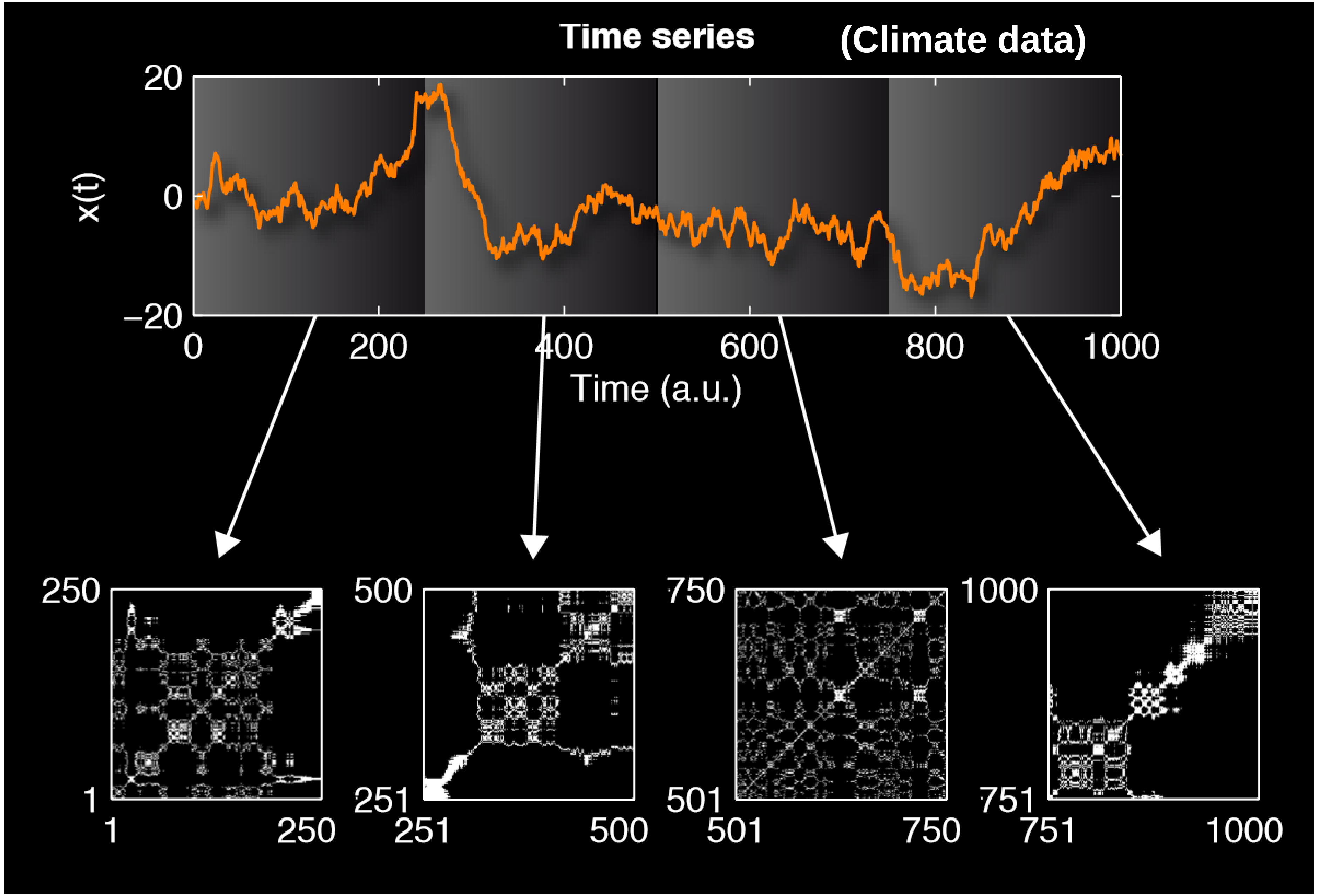


Logistic function



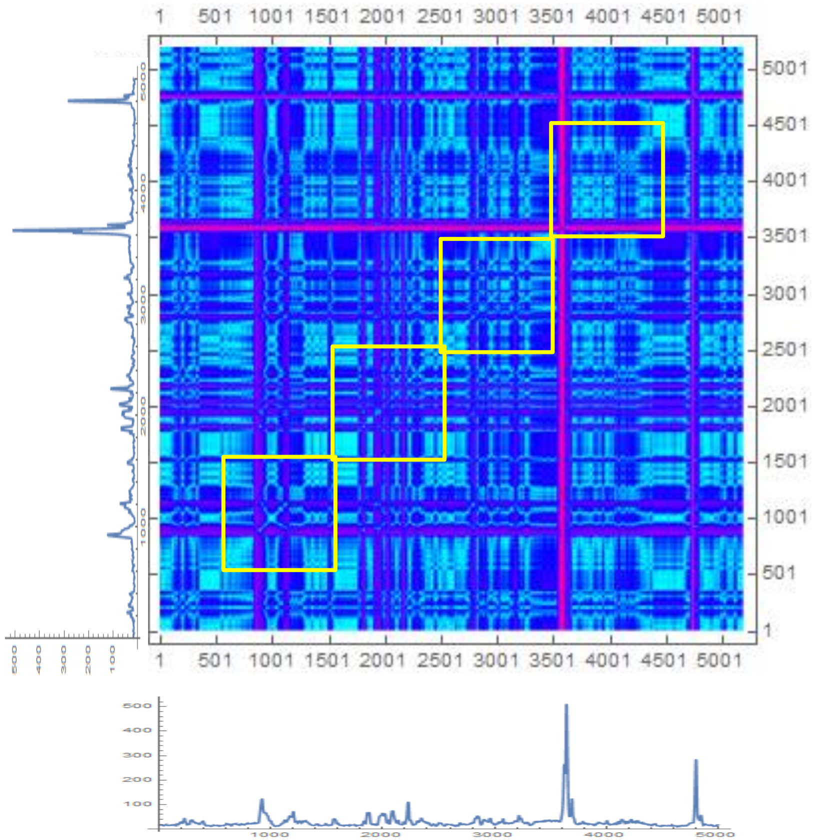
Detection of transitions and changes in behaviour.

The vertical zones are an expression of the *laminarity* of the system and indicate chaotic transitions



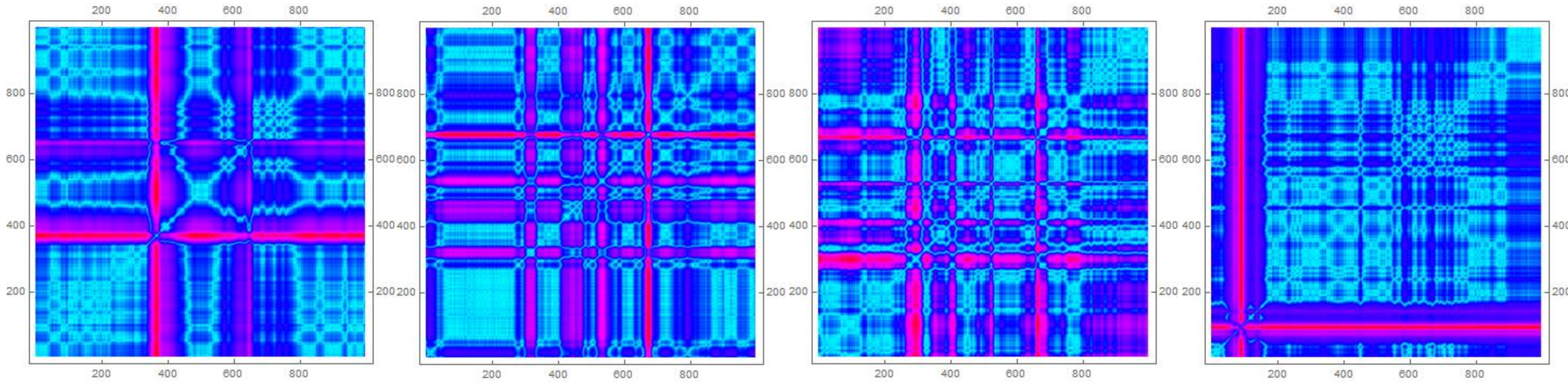
A sliding window enables classification and quantification of various kinds of behaviour

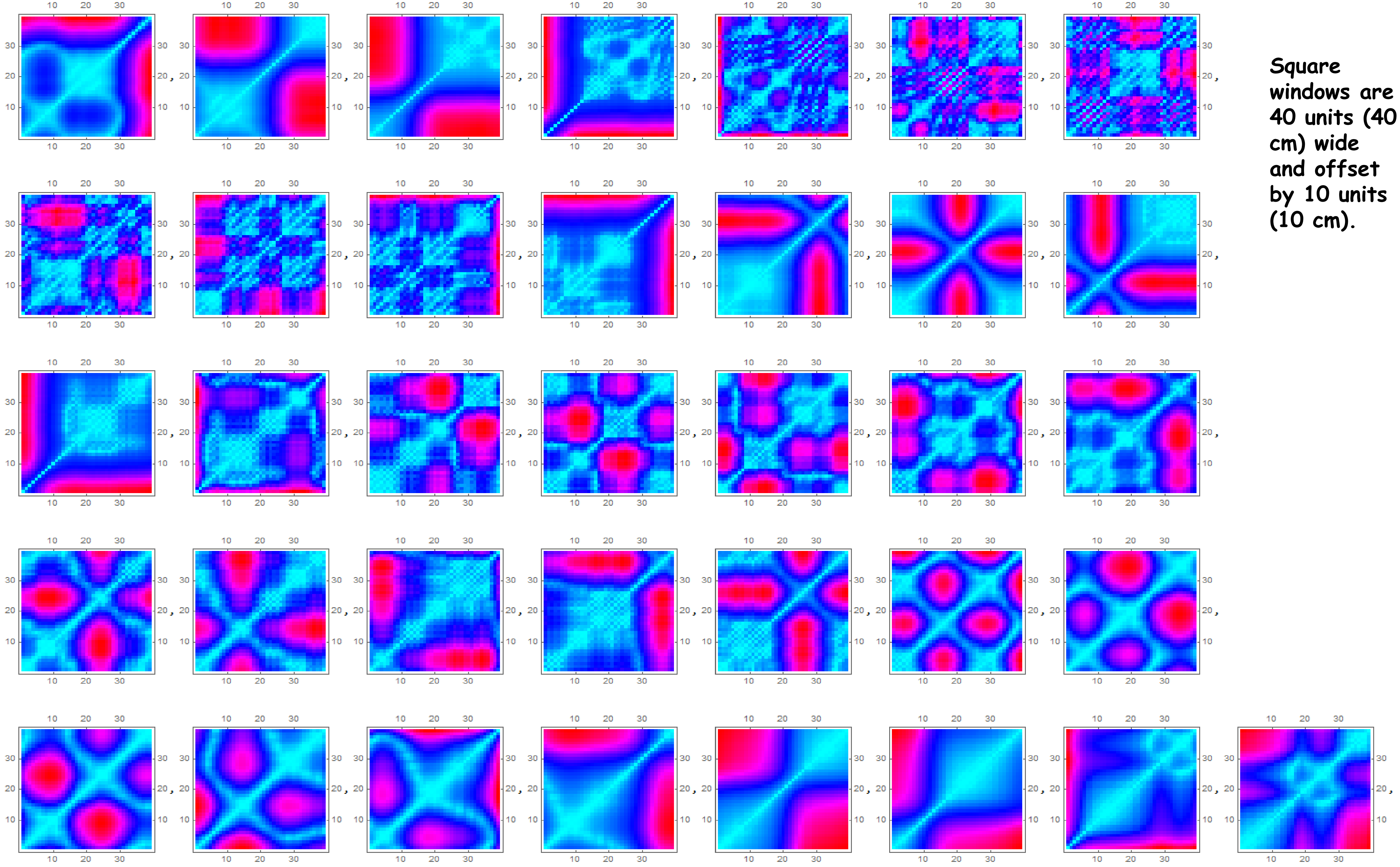
From Marwan, 2005



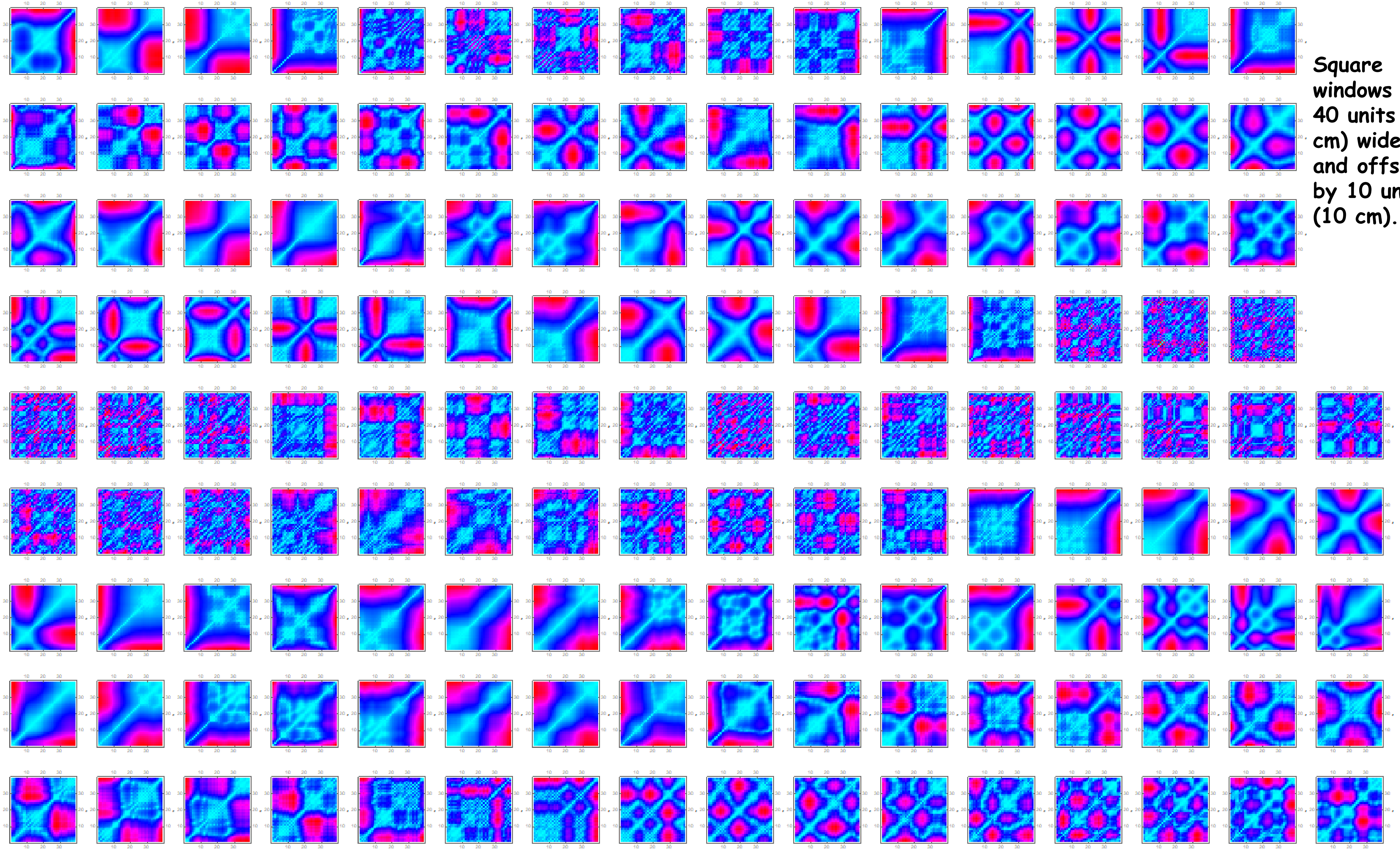
Square windows are 1000 units (10 m) wide and offset by 1000 units (10 m).

A sliding window enables classification and quantification of various kinds of behaviour





Square windows are 40 units (40 cm) wide and offset by 10 units (10 cm).

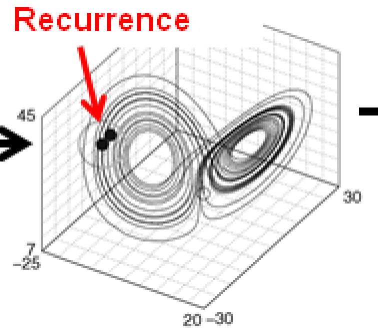


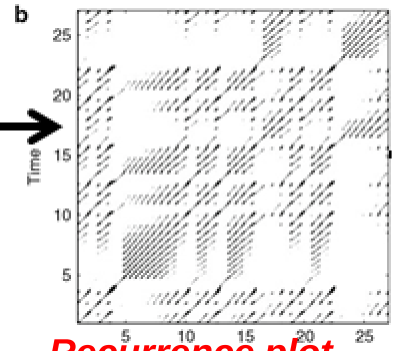
Square windows are 40 units (40 cm) wide and offset by 10 units (10 cm).

Recurrence plots visualise the trajectories of system states in phase space and the probability that given states recur within a given tolerance

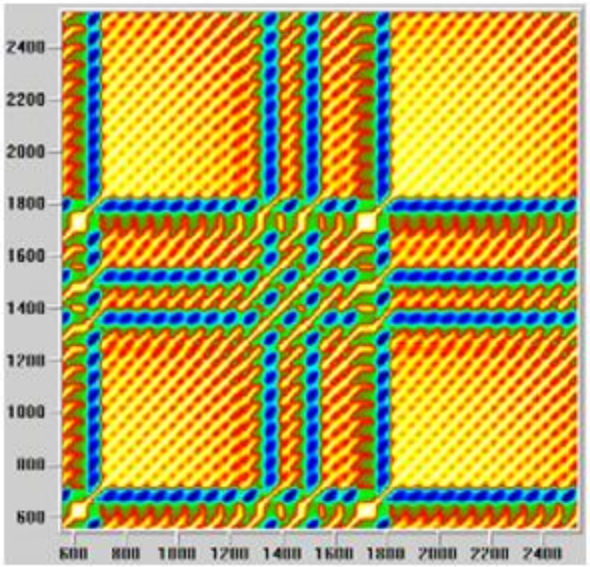
Coupled sets of nonlinear processes leads to chaotic behaviour in space and time

The behaviour of such systems can be represented by an attractor that is a plot of all possible states of the system

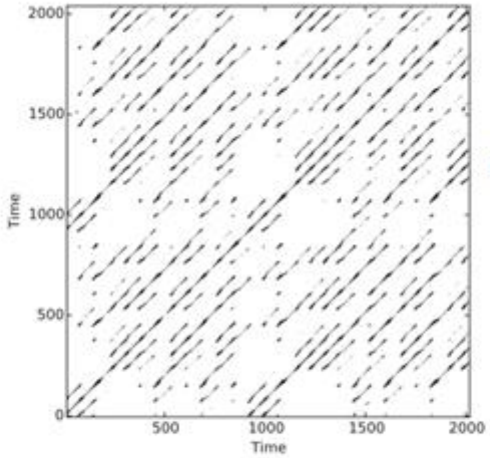




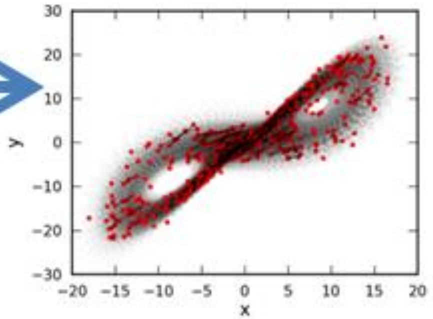
Recurrence plot



Observed



Lengths of diagonal lines is a measure of predictability



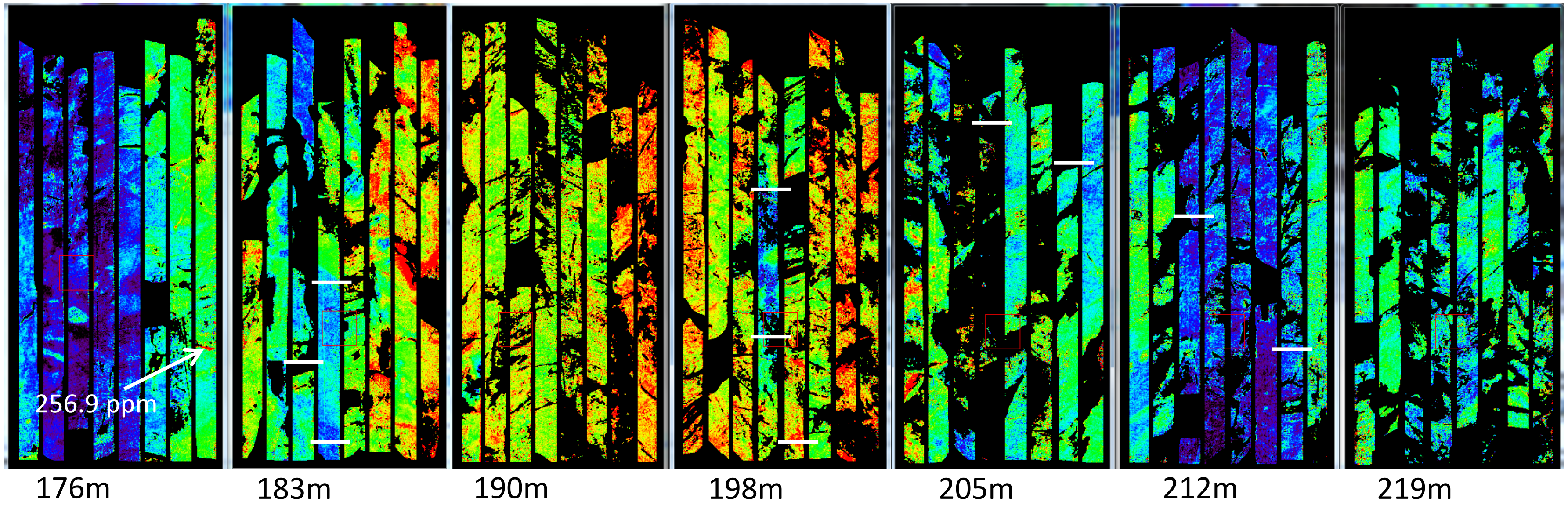
The recurrence plot can be converted to a recurrence network that mimics the attractor for the system

The Data

Mineralogy / Chemistry

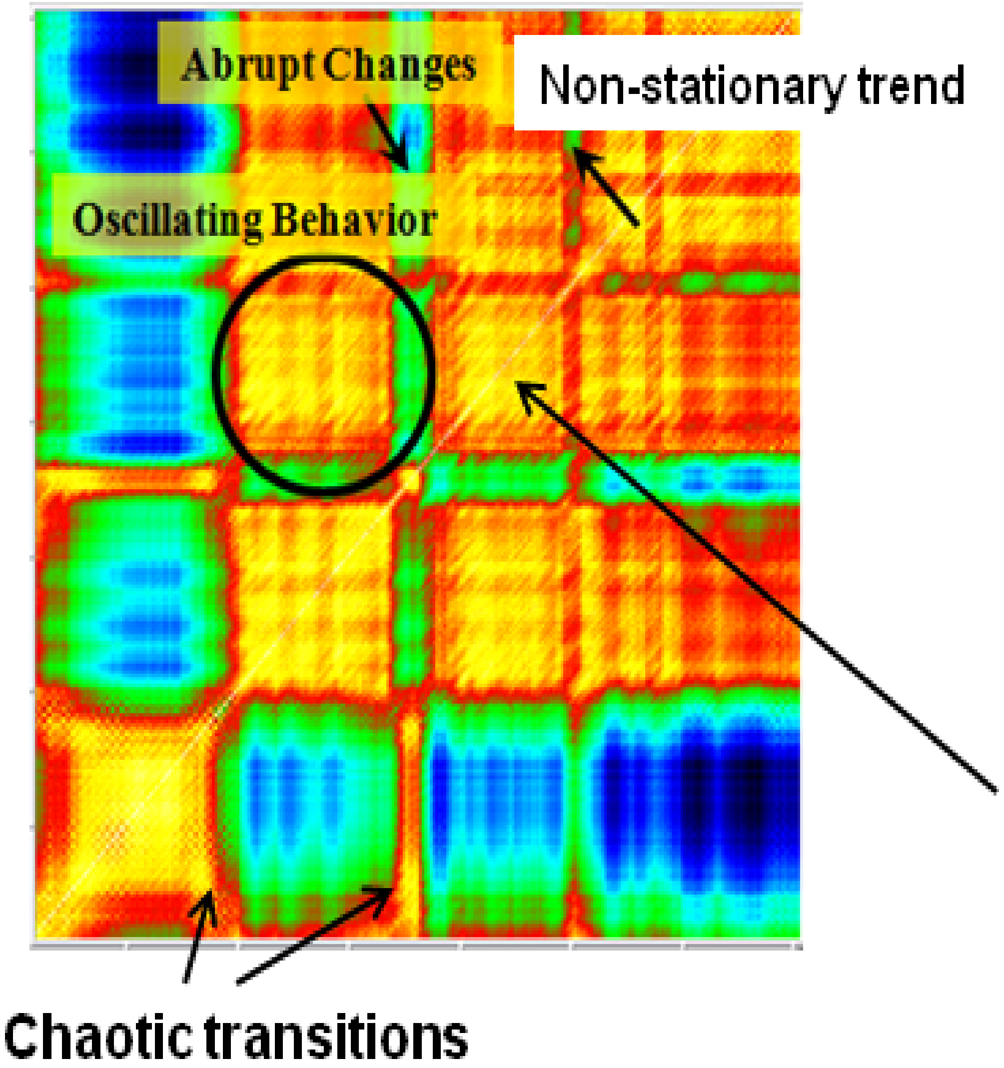
Sericite composition

Colour stretch: 2190-2000nm (Blue-red)



Near infra-red reflectance spectra of many kms of diamond drill core → detailed mineralogy + chemistry at mm / micron resolution

The colour → chemical composition of white micas from K-rich to Fe-rich



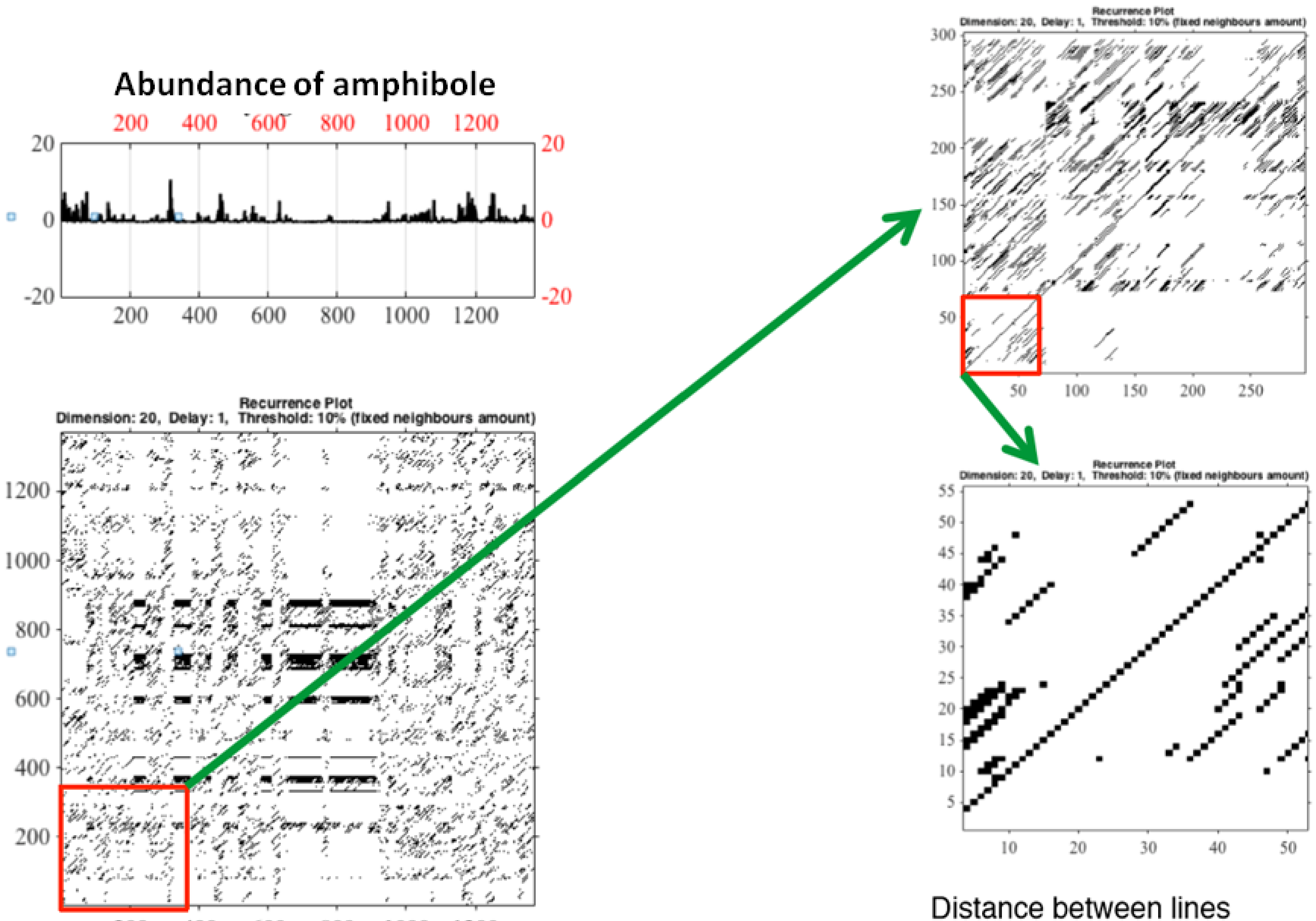
Diagonal lines: determinism, oscillatory behaviour

Vertical and horizontal lines: Abrupt changes, chaotic or periodic transitions

Fading pattern along a diagonal: Non-stationary trend.

Distances between diagonal lines: Recurrence of states.

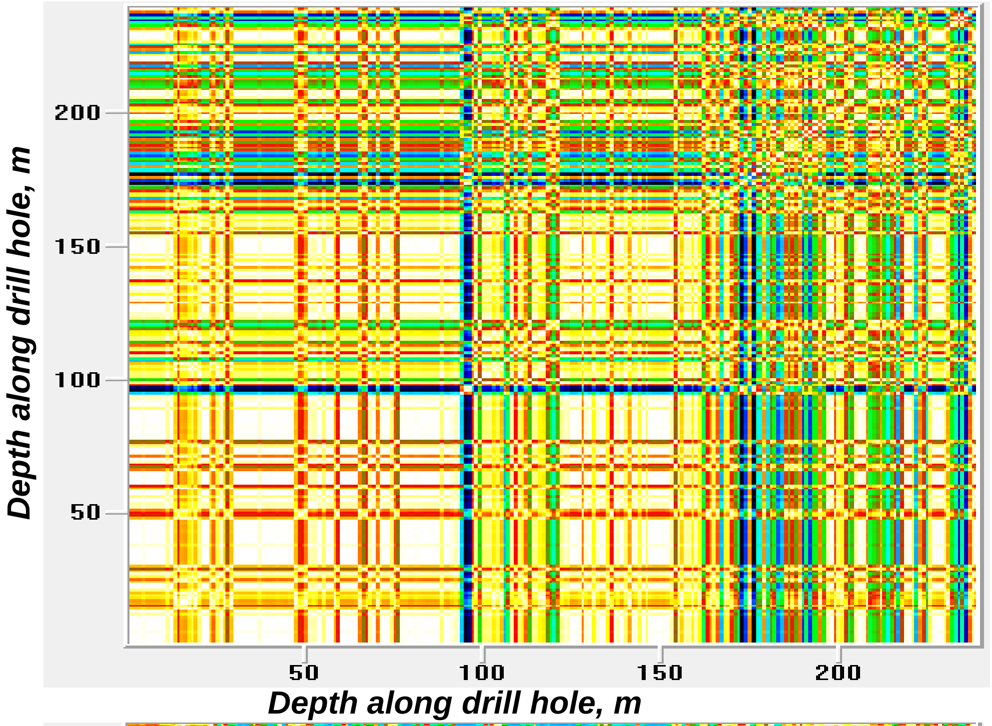
Interpretation of recurrence plots. Quantification metrics include determinism (histogram of lengths of diagonal lines), laminarity (histogram of vertical or horizontal distances between diagonal lines), trapping frequency of chaotic transitions (frequency of vertical and horizontal lines) and autocorrelation spectra of recurrence states.



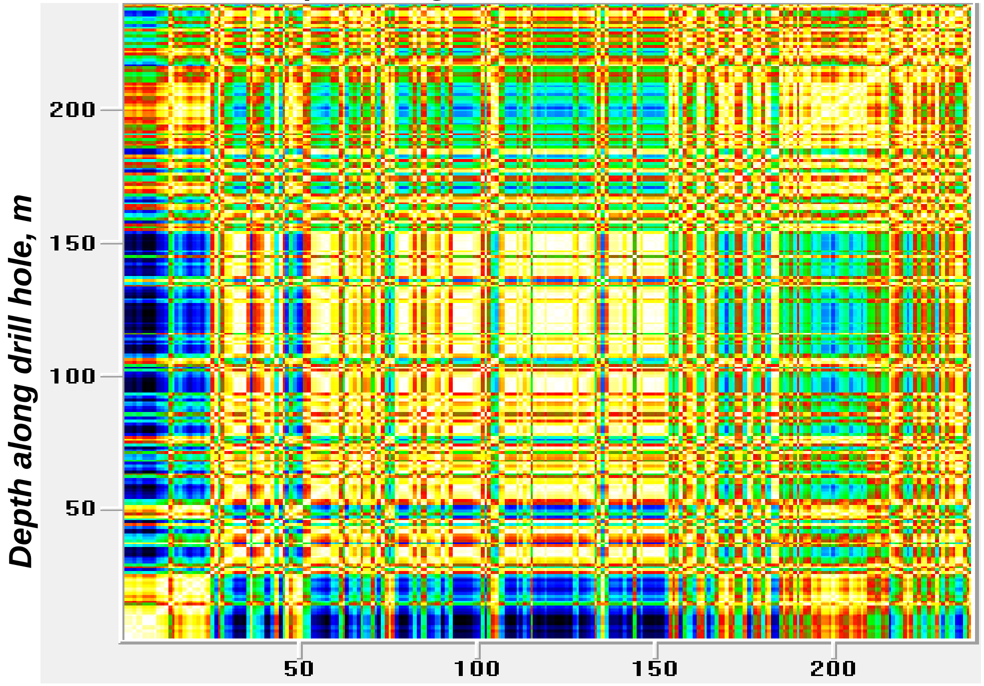
Deterministic

Distance between lines
→ Frequency = 9-14i increments

Sericite recurrence plot



Chlorite recurrence plot



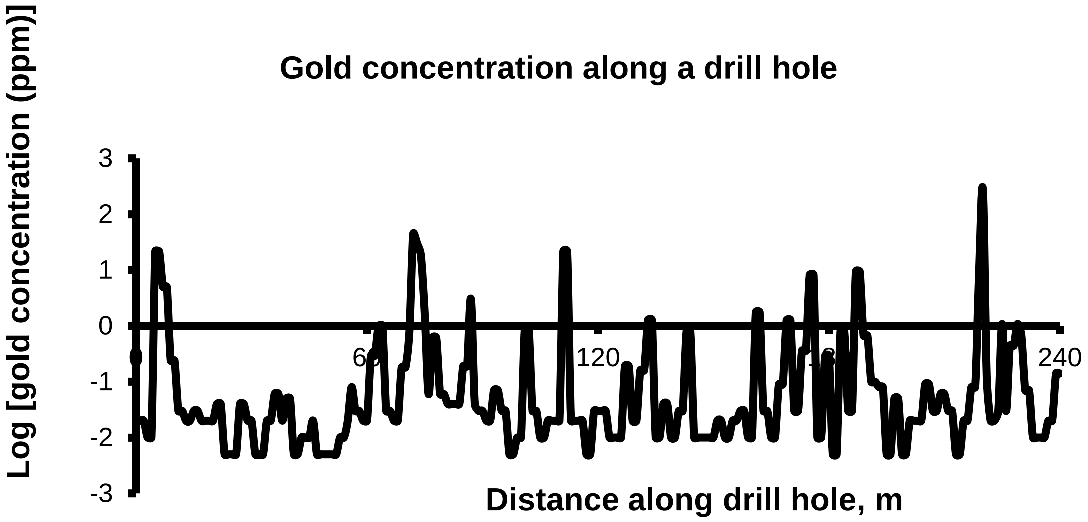
Strong laminarity in both.

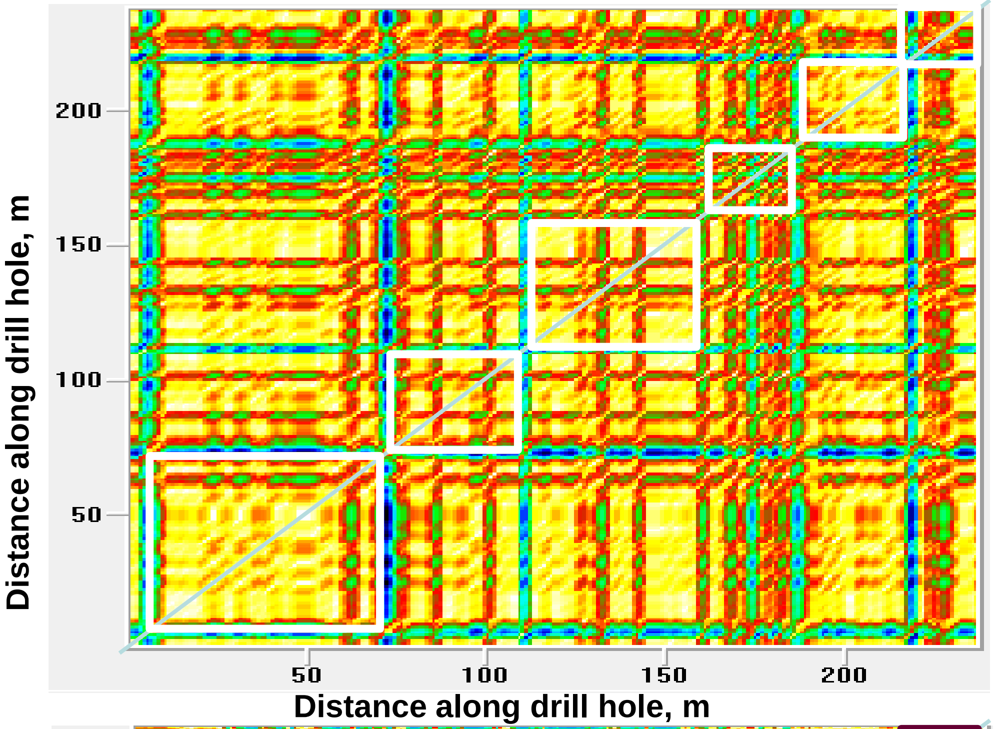
Evidence of many chaotic transitions.

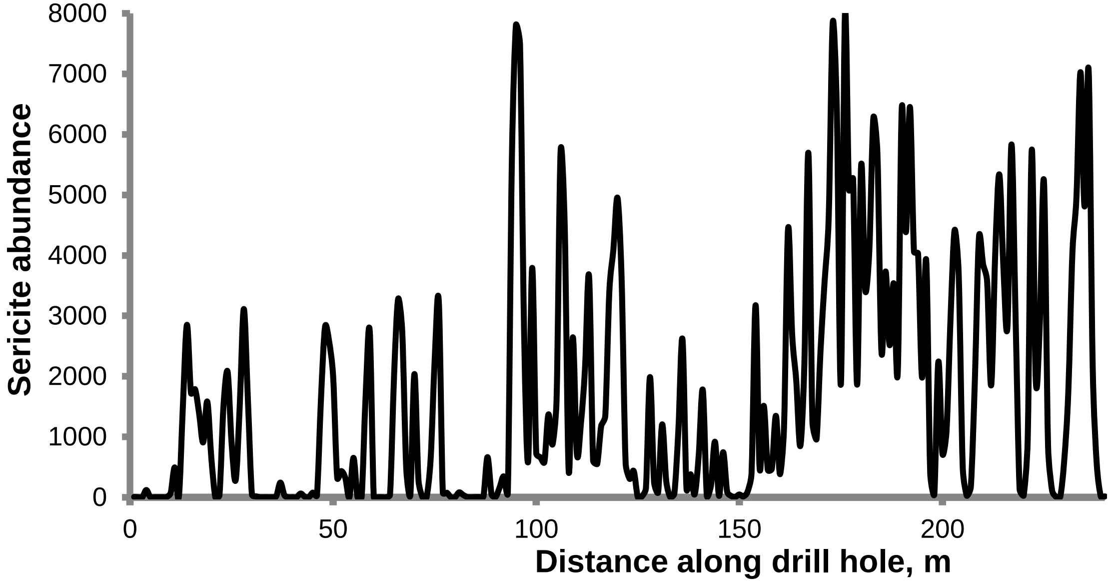
However the correlation in space between chaotic transitions is low.

What about correlations with gold?

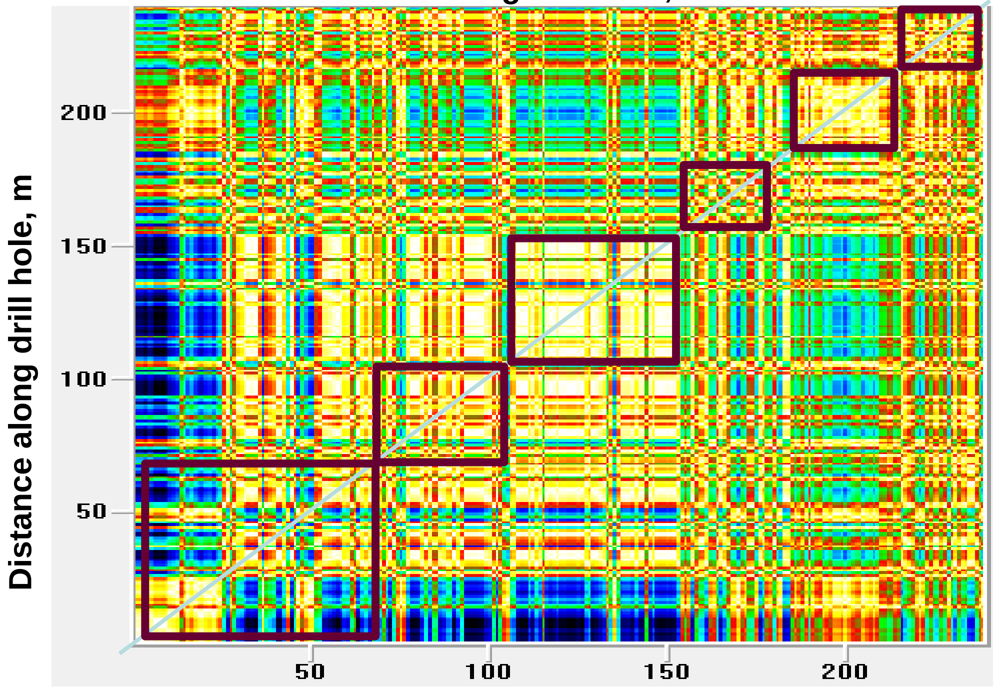
Gold concentration along a drill hole



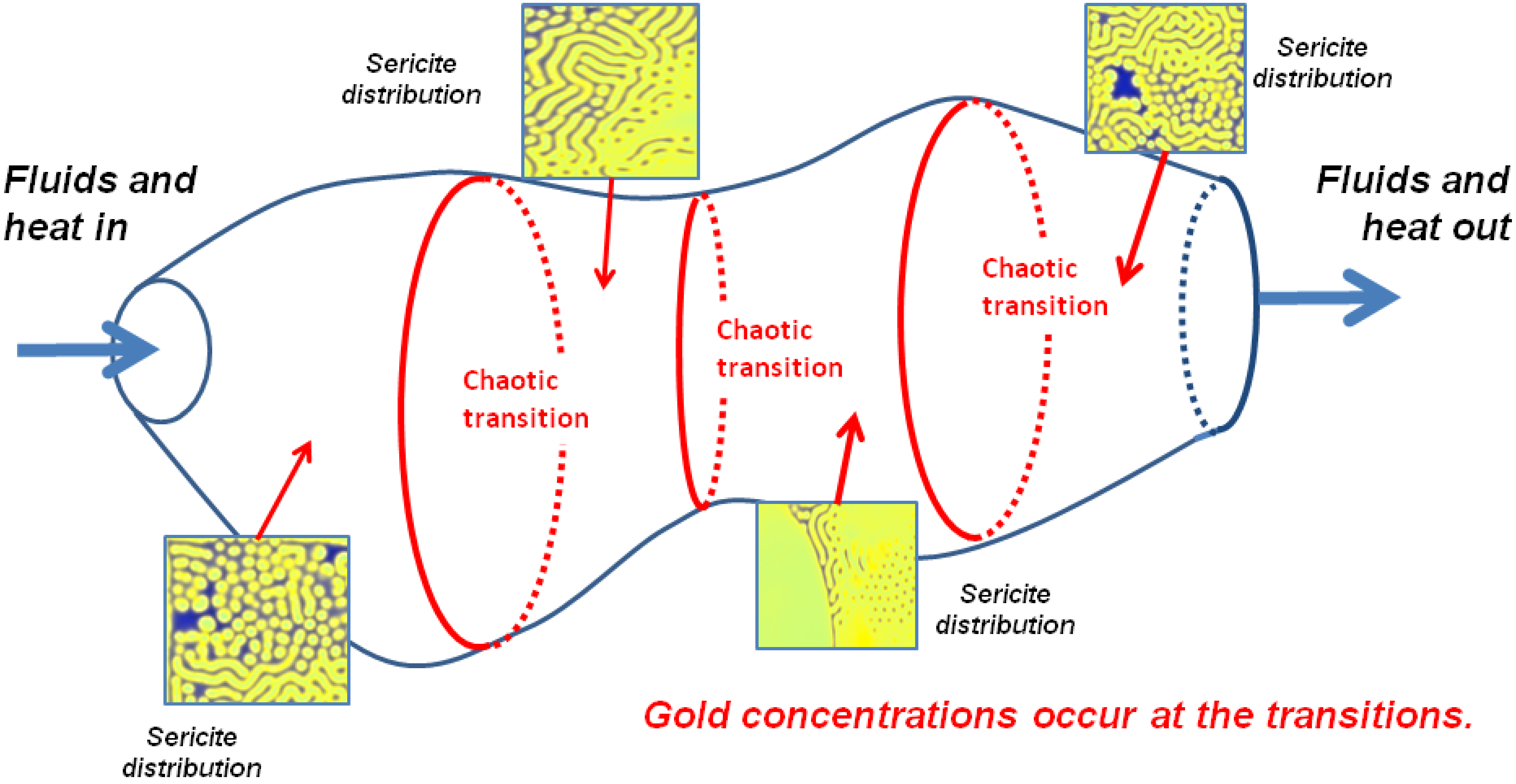




Sericite abundance along a drill hole

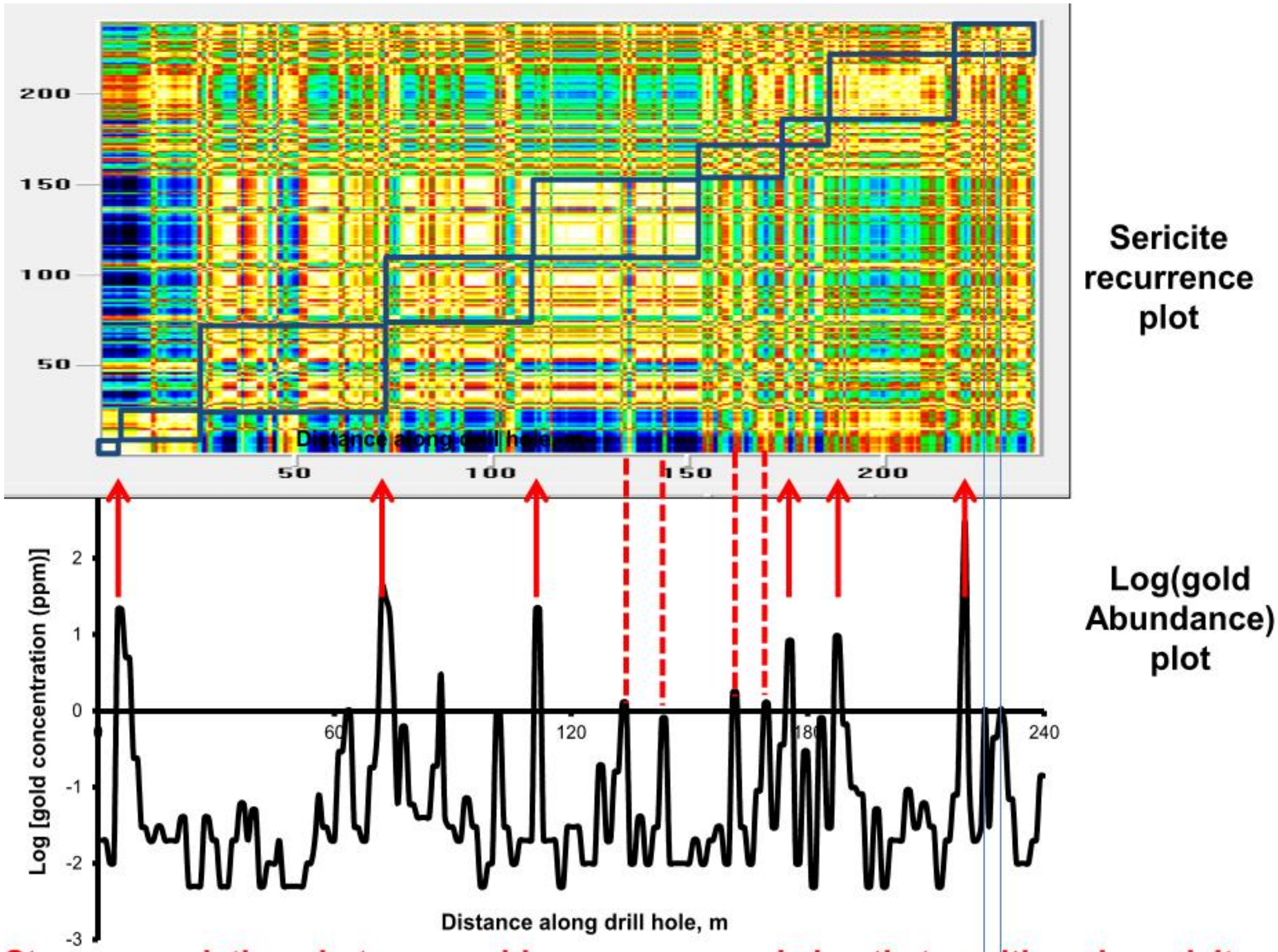


Chaotic transitions within an ore body



Can we map these transitions?

Produce a bar code or “magnetic stripe stratigraphy” for the ore system



Strong correlations between gold occurrence and chaotic transitions in sericite

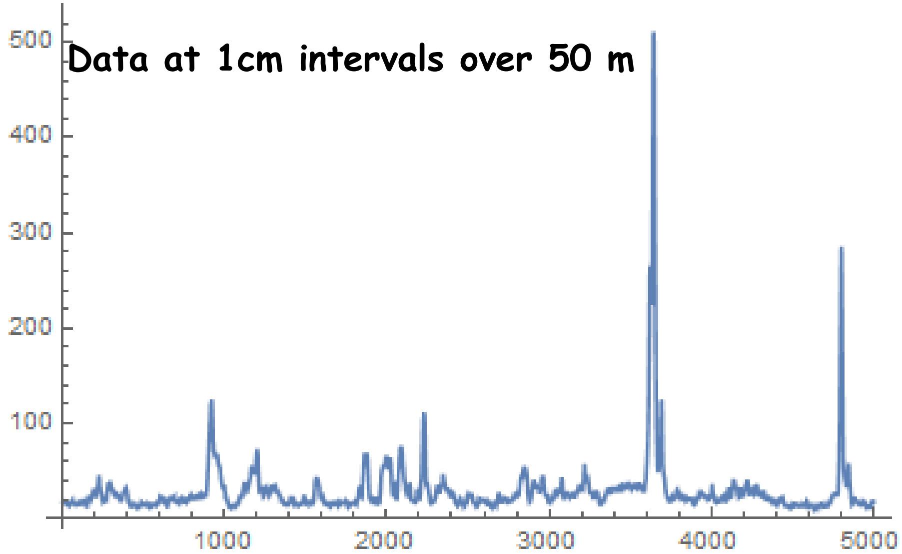
Cross correlation between sericite recurrence patterns and gold concentration.

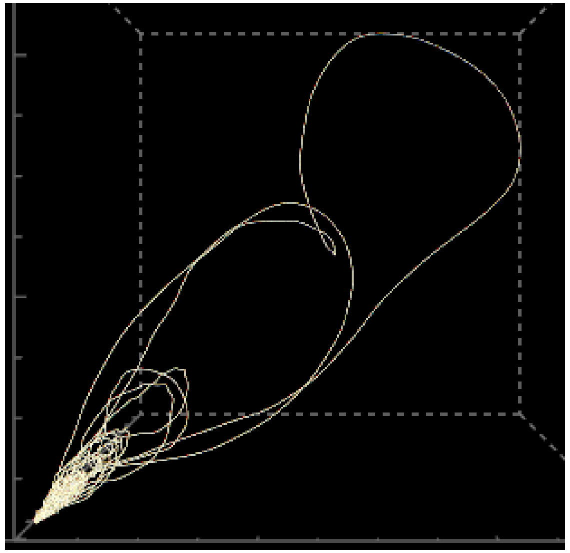
The top panel shows a recurrence plot for sericite in a gold deposit from the Yilgran of Western Australia.

The lower panel show the strong correlation between high values of gold and chaotic transitions in the sericite data set.

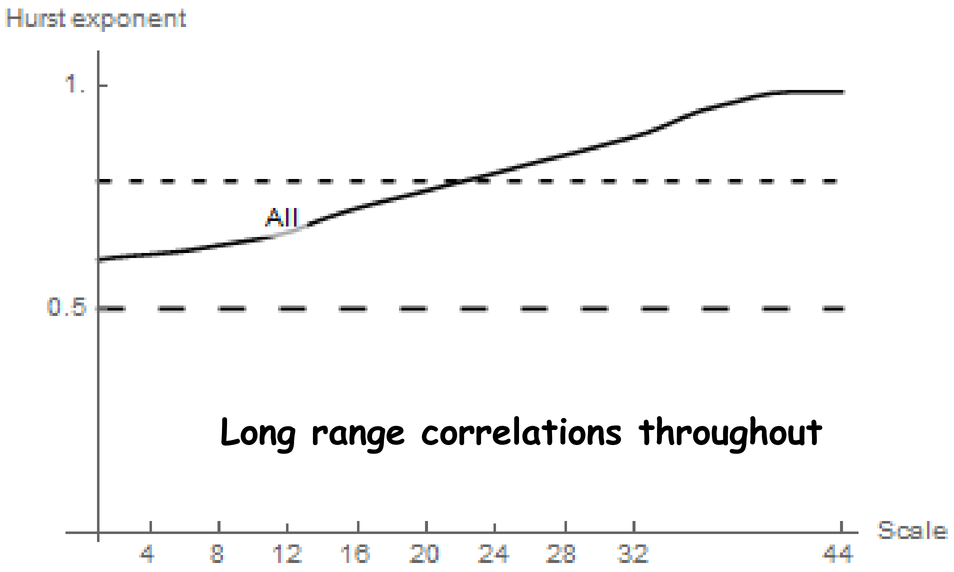
It is proposed that algorithms that incorporate these kinds of cross correlations be combined with algorithms used for prediction in order to improve the range of prediction in these kinds of systems.

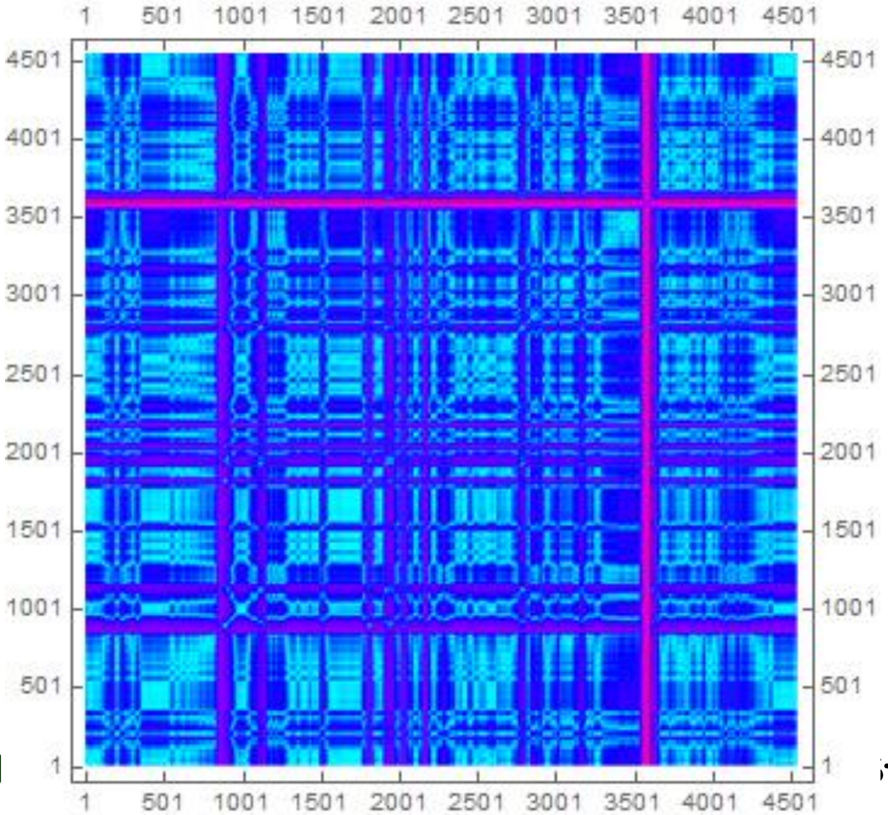
Does this help us predict?



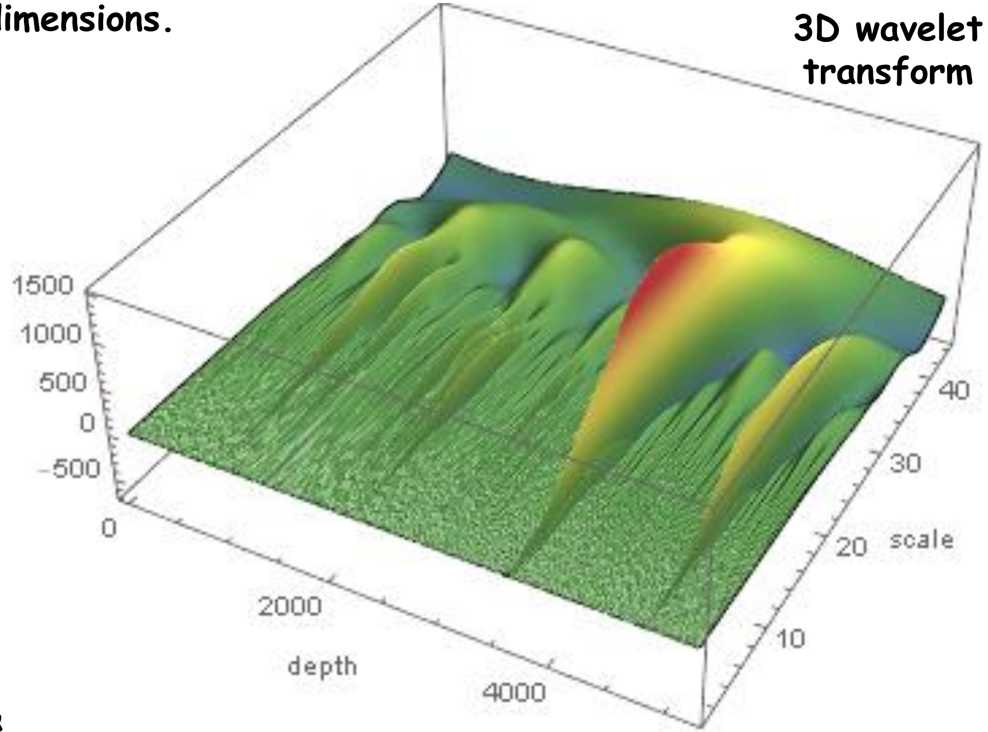


Spatial attractor within embedding dimension of 4 projected on to 3 dimensions.





Recurrence plot

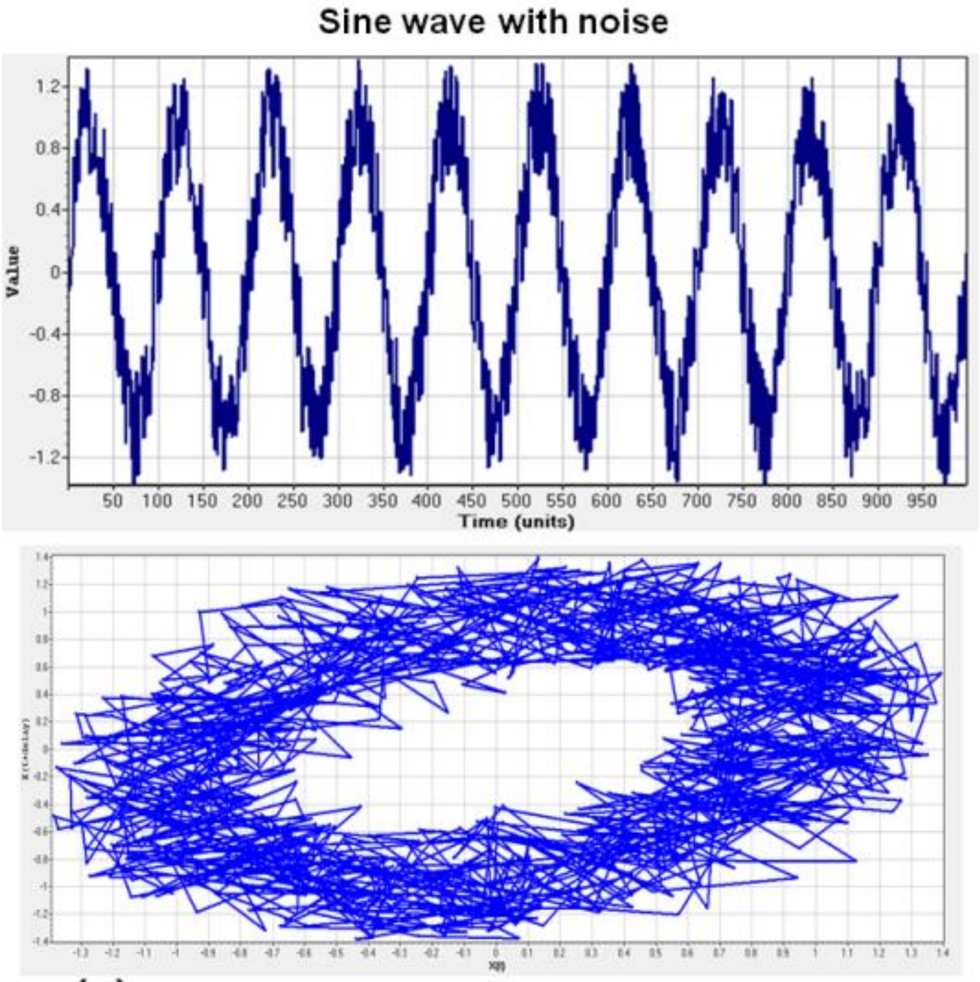


3D wavelet transform

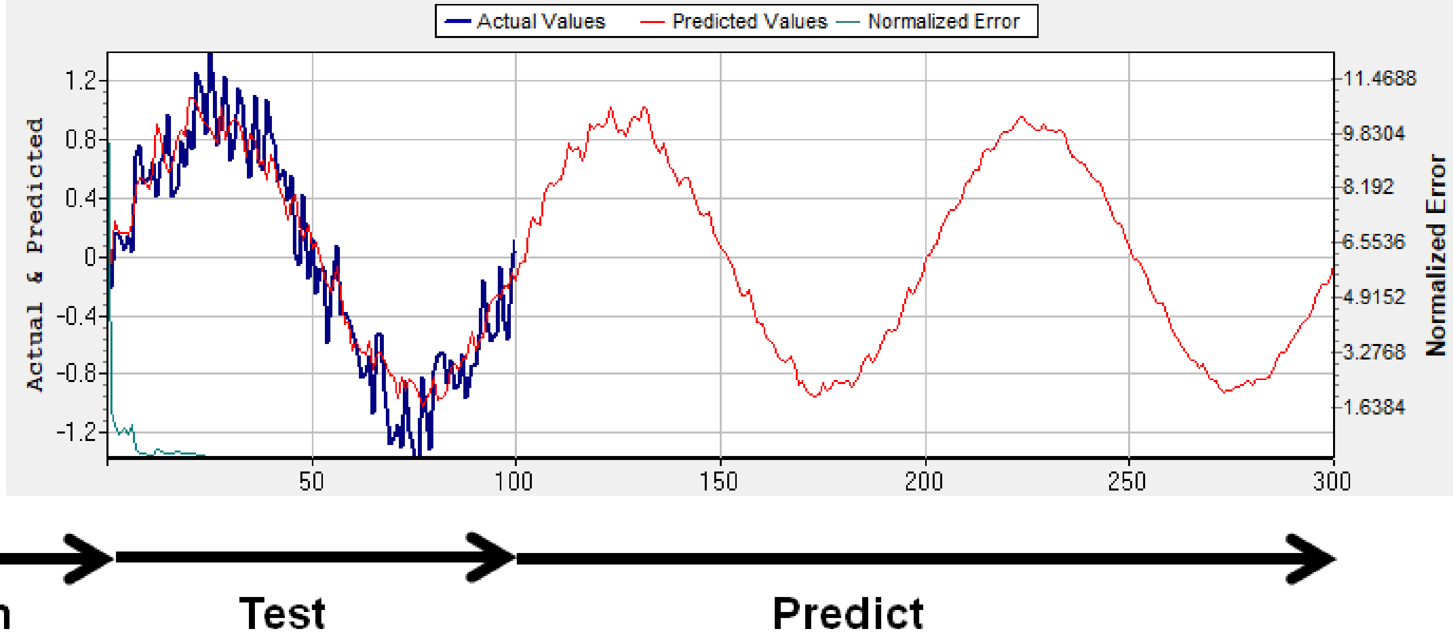
Kalgoorl

tructural Geology &

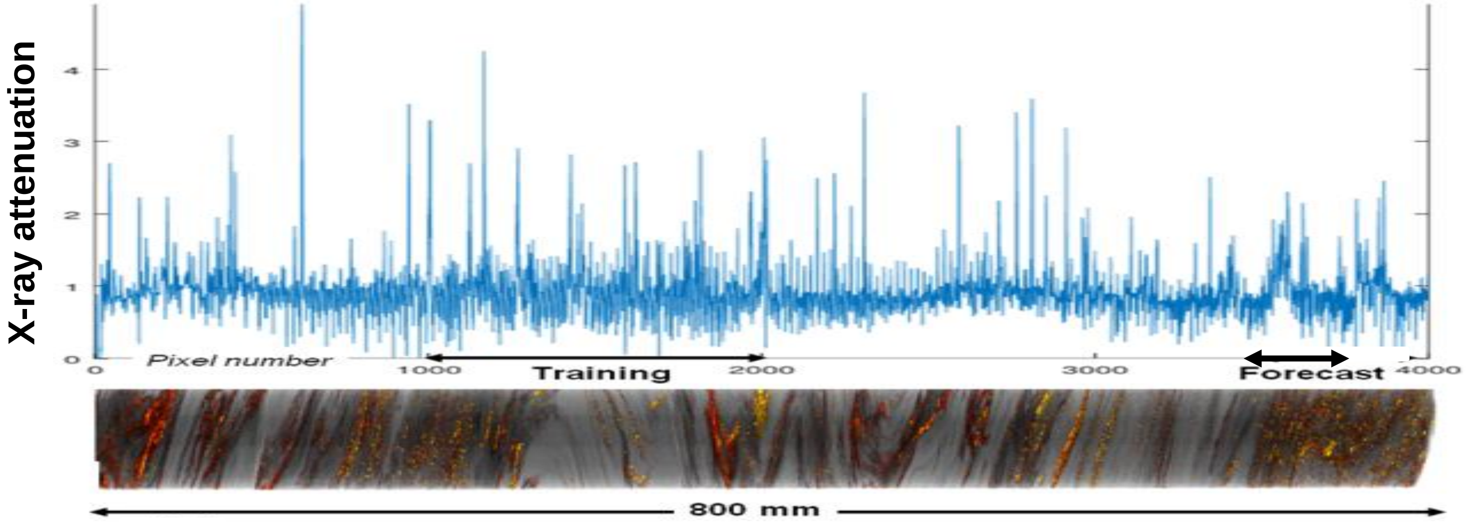
Prediction of a sine wave signal with noise

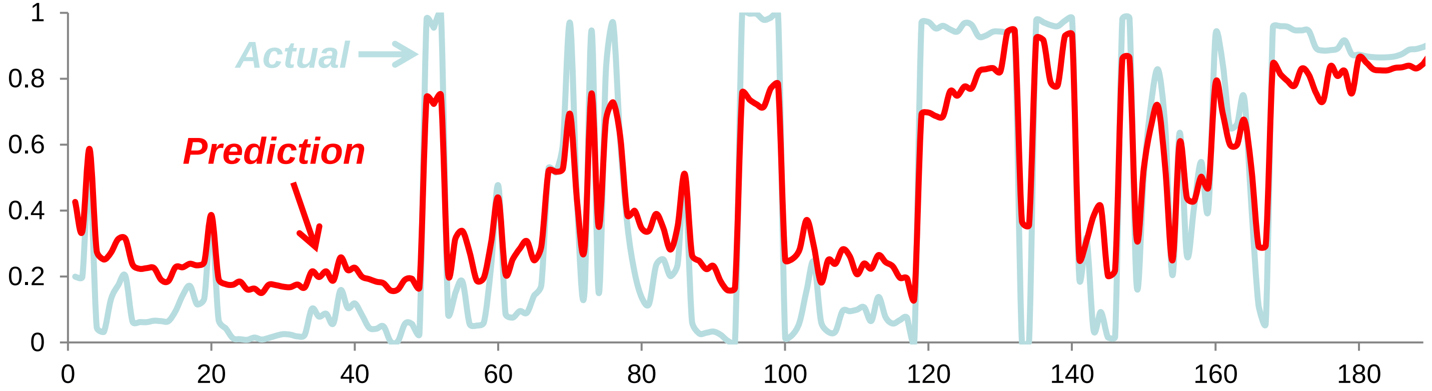


(a) Attractor



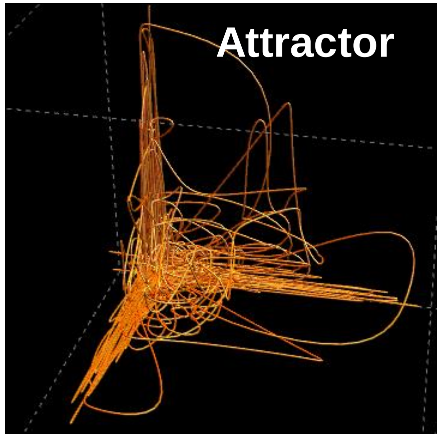
(b)

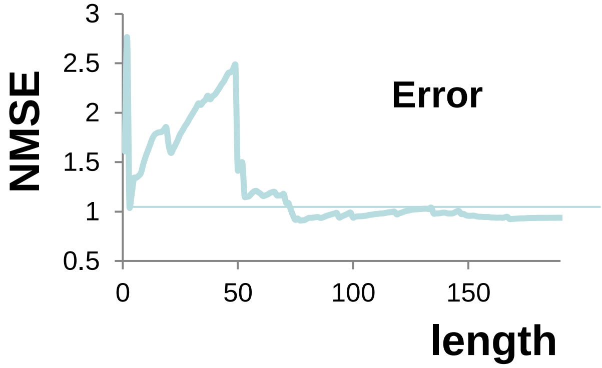




Prediction. X-ray attenuation.

OREXPLORE





Software links:

Wavelet analysis, MMWT-method, singularity spectrum:
LastWave: http://www.cmap.polytechnique.fr/~bacry/LastWave/ **This runs well in Windows**

Recurrence plots and quantitative recurrence analysis:
http://www.recurrence-plot.tk/ **This is the font of all knowledge**
http://tocsy.pik-potsdam.de/CRPtoolbox **Requires MATLAB**
http://web.archive.org/web/20070131023353/http://www.myjavaserver.com/~nonlinear/vra/download.html **This runs well in Windows**

Recurrence networks and quantitative recurrence analysis:
http://tocsy.pik-potsdam.de/pyunicorn.php **Runs on a range of platforms**